

Problem Sheet 10  
DRE 7007 Mathematics

**Problems**

1. We consider the function  $f : D \rightarrow \mathbb{R}$  on  $D = (1, \infty) \subseteq \mathbb{R}$ , given by

$$f(x) = \frac{1}{2} \left( x + \frac{a}{x} \right)$$

where  $a \in \mathbb{R}$  is a given number.

- a) Show that  $f$  defines an operator  $f : D \rightarrow D$  if  $a \in (1, 3)$ .
- b) Show that  $f$  is a contraction and find its fixed point for each  $a \in (1, 3)$ . What about  $a = 1$  and  $a = 3$ ?
- c) Is  $D$  complete?

2. Consider the correspondence  $F : [0, 2] \rightrightarrows [0, 2]$  given by

$$F(x) = \begin{cases} \{2\} & x \in [0, 1) \\ \{0, 2\} & x = 1 \\ \{0\} & x \in (1, 2] \end{cases}$$

Describe the graph of  $F$ . Does Kakutani's Theorem apply? Does  $F$  have any fixed points?

**Keep answers as short and to the point as possible. Answers must be justified.**