BI NORWEGIAN BUSINESS SCHOOL

Written examination:	**DRE 70171 Mathematics, Ph.D.**	
Examination date:	09/2014	Total no. of pages: 2
Permitted examination	A bilingual dictionary and BI-approved calculator TEXAS	
support material:	INSTRUMENTS BA II Plus	
Answer sheets:	Squares	
	Counts 100% of DRE 7017	The subquestions have equal weight
Mock exam		Responsible department: Economics

All subquestions have equal weight. All answers should be justified.

Question 1.

We consider the linear system of differential equations given by

$$\dot{x} = -5x + y + 1$$
$$\dot{y} = 2x - 4y + 14$$

(a) Find the steady state $(\overline{x}, \overline{y})$.
(b) Rewrite the systen in the form $\mathbf{z}' = A\mathbf{z}$ and use this to solve the system.
(c) Is the system stable? Find (x, y) at $t = 200$ in terms of the initial state (x_0, y_0).

Question 2.

We consider the function $f(x, y, z, w) = xw - yz$ defined on $\mathbb{R}^4$.

(a) If f convex? Is it concave?
(b) Find the global maximum and minimum values of f, if they exist.

Question 3.

We consider the function $f(x, y, z) = x^2 + y^2 + z^2$ with domain $D = \{(x, y, z) : 2x^2 + 6y^2 + 3z^2 \geq 36\}$, and the optimization problem

$$\min_{(x,y,z)\in D} f(x, y, z)$$

(a) Is the set D compact?
(b) Is f convex? Is it concave?
(c) Solve the optimization problem.

Question 4.

We consider the optimal control problem

$$\max \sum_{t=0}^{T} (3 - u_t) x_t^2 \quad \text{subject to} \quad x_{t+1} = x_t u_t,\ x_0 = 1,\ u_t \in U$$

with control region $U = [1, 3] \subseteq \mathbb{R}$.

(a) Solve the problem for $T = 3$ using dynamical programming.
(b) Solve the problem for a general T.