Regnet Eksamen i Met 22141

Gitt 19.05.10

Oppgave 4 og Oppgave 5

Regnet Eksamen i Met 22141.

Gitt 12.06.06

Ross Oppgave 3.10

X og Y er uavhengige

$f(x,y) = f_X(x) \cdot f_Y(y)$

Herav følger

$$f_{X|Y}(x,y) = \frac{f(x,y)}{f_Y(y)} = \frac{f_X(x) \cdot f_Y(y)}{f_Y(y)} = f_X(x)$$

$$E(X \mid Y=y) = \int_{-\infty}^{\infty} x\, f_{X|Y}(x \mid y)\, dx =$$

$$\int_{-\infty}^{\infty} x \cdot f_X(x)\, dx = E(X).$$

Ross Oppgave 3.12 Side 163.

$$f(x,y) = \frac{e^{-\frac{x}{y}} e^{-y}}{y} \qquad \begin{array}{l} 0 < x < \infty \\ 0 < y < \infty \end{array}$$

Vis at $E(X \mid Y=y) = y$.

Finner først:

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_0^{\infty} \frac{e^{-\frac{x}{y}} e^{-y}}{y}\,dx =$$

$$e^{-y} \cdot \int_0^{\infty} \frac{e^{-\frac{x}{y}}}{y}\,dx \cdot$$

Setter vi $u = \frac{x}{y}$ vil $du = \frac{1}{y}\,dx$

$$\int e^{-\frac{x}{y}} \cdot \frac{1}{y}\,dx = \int e^{-u}\,du = -e^{-u} + c = -e^{-\frac{x}{y}} + c$$

$$\int_0^{\infty} \frac{e^{-\frac{x}{y}}}{y}\,dx = \left[-e^{-\frac{x}{y}}\right]_0^{\infty} = 1$$

Herav følger $f_Y(y) = e^{-y}$

$$f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)} = \frac{e^{-\frac{x}{y}} \cdot e^{-y}}{y e^{-y}} = \frac{e^{-\frac{x}{y}}}{y}.$$

$$E(X \mid Y=y) = \int_0^{\infty} \underset{u}{x} \cdot \underset{v'}{\frac{e^{-\frac{x}{y}}}{y}}\,dx = \left[\underset{u}{x}\underset{v}{\left(-e^{-\frac{x}{y}}\right)}\right]_0^{\infty} - \int_0^{\infty} \underset{u'}{1}\underset{v}{\left(-e^{-\frac{x}{y}}\right)}dx =$$

$$\int_0^{\infty} e^{-\frac{x}{y}}\,dx = \left[-e^{-\frac{x}{y}} \cdot y\right]_0^{\infty} = -0 + e^0 y = y$$