FORELESNING 17 ELE 3719

EIVIND ERIKSEN MAR 16 2012 MATEMATIKK VF

PLAN:

1. LINEÆR REGRESJON [MKF] 2.4
2. EKSAMENSOPPGAVER

ELE 11/2011 Oppg. 3-4
ELE 06/2011 Oppg. 1-2

} Regnet på tavlen – Se LF i eksamensoppgave-samling

1. Lineær regresjon

– en uavh. variabel x_1 | – n uavh. variable $x_1 \ldots x_n$

Modell:

$$y = \beta_0 + \beta_1 x_1$$

x_1	y
x_{11}	y_1
x_{12}	y_2
$\vdots$	$\vdots$

$$\boxed{\begin{aligned} y_1 &= \beta_0 + \beta_1 x_{11} + \varepsilon_1 \\ y_2 &= \beta_0 + \beta_1 x_{12} + \varepsilon_2 \\ &\vdots \end{aligned}}$$

$$\underline{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_N \end{pmatrix} \quad X = \begin{pmatrix} 1 & x_{11} \\ 1 & x_{12} \\ \vdots & \\ 1 & x_{1N} \end{pmatrix} \quad \underline{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix}$$

$$\underline{y} = X \cdot \underline{\beta} + \underline{\varepsilon} \qquad \underline{\varepsilon} = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_N \end{pmatrix}$$

Feil ved minste kvadraters metode

$$E = \varepsilon_1^2 + \ldots + \varepsilon_N^2 = \underline{\varepsilon}^T \underline{\varepsilon} = (\underline{y} - X\underline{\beta})^T (\underline{y} - X\underline{\beta})$$

$$= \underline{y}^T \underline{y} - \underline{y}^T X \underline{\beta} - \underline{\beta}^T X^T \underline{y} + \underline{\beta}^T X^T X \underline{\beta}$$

$$= \underline{\beta}^T (X^T X) \underline{\beta} - 2 \underline{y}^T X \cdot \underline{\beta} + \underline{y}^T \underline{y}$$

Modell:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_n x_n$$

x_1	x_2	$\ldots$	x_n	y
x_{11}	x_{21}		x_{n1}	y_1
x_{12}	x_{22}	$\ldots$	x_{n2}	y_2
$\vdots$	$\vdots$		$\vdots$	$\vdots$

$$\boxed{\underline{y} = X \cdot \underline{\beta} + \underline{\varepsilon}}$$

$$\underline{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \end{pmatrix} \quad X = \begin{pmatrix} 1 & x_{11} & x_{21} & \ldots \\ 1 & x_{12} & x_{22} & \ldots \\ \vdots & \vdots & \vdots & \\ 1 & x_{1N} & x_{2N} & \ldots \end{pmatrix}$$

$$\underline{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_n \end{pmatrix}$$

Resultater som for $\underline{n=1}$.

Vi må minimere

$$E = \underline{\beta}^T \underset{\underset{A}{\shortparallel}}{(X^T X)} \underline{\beta} - \underset{\underset{B}{\shortparallel}}{2\underline{y}^T X} \cdot \underline{\beta} + \underline{y}^T \underline{y}$$

med hensyn til $\underline{\beta}$.

$$\frac{\partial}{\partial \underline{x}} \left(\underline{x}^T A \underline{x} \right) = 2A \cdot \underline{x}$$

$$\frac{\partial}{\partial \underline{x}} \left(B \cdot \underline{x} \right) = B^T$$

Stasjonære pkt:

$$\frac{\partial E}{\partial \underline{\beta}} = 2A \cdot \underline{\beta} + B^T$$

$$= 2X^T X \cdot \underline{\beta} - 2X^T \underline{y} = \underline{0}$$

$$2X^T X \cdot \underline{\beta} = 2X^T \underline{y}$$

$$(X^T X) \cdot \underline{\beta} = X^T \cdot \underline{y}$$

Anta $|X^T X| \neq 0$: $\boxed{\underline{\beta} = (X^T X)^{-1} \cdot X^T \underline{y}}$

Er dette et minimum: JA.

$\underline{\beta}$ minimum $\Longleftrightarrow$ $A = X^T X$ er positiv definit

Sjekk: $\underline{\beta}^T (X^T X) \underline{\beta} \geq 0$ for alle $\underline{\beta}$, og $>$ hvis $\underline{\beta} \neq \underline{0}$.

$$\shortparallel$$

$$(X\underline{\beta})^T \cdot (X\underline{\beta})$$

$$\shortparallel$$

$$(h_1 \ldots h_N) \cdot \begin{pmatrix} h_1 \\ h_2 \\ \vdots \\ h_N \end{pmatrix} = h_1^2 + h_2^2 + \ldots + h_N^2 \geq 0$$

* dette betyr at $X^T X$ er positiv semidefinit
* siden $|X^T X| \neq 0$, så er $X^T X$ positiv definit