FORELESNING 17

ECE 3717

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PLAN:

① Repetisjon: Diff. likninger som kan løses ved
– direkte integrasjon – separable diff. likninger

② Lineære første ordens diff. likninger og
integrerende faktor.

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Generelt om diff. likninger

ordinære – funksjoner i én variabel
derivasjon mht én variabel

partielle – funksjoner i flere variable
partielle deriverte i diff. likningen } ikke pensum

orden : ordenen til en diff. likning = høyeste
ordens derivert som inngår i diff. likningen

Eks:
$$\left.\begin{aligned} y' &= te^t \\ y' &= y - y^2 \end{aligned}\right\} \text{første ordens}$$

$$y'' - 2y' + 3y = 0 \qquad \text{andre ordens}$$

① Repetisjon:

Eks: $y' = te^t$ — Kan løses direkte ved integrasjon

$$y = \int \underset{u}{t}\,\underset{v'}{e^t}\,dt$$

$$= u \cdot v - \int u' v \, dt$$

$$= t \cdot e^t - \int 1 \cdot e^t \, dt$$

$$y = te^t - e^t + C$$

Eks: $y' = y - y^2 = (y-y^2) \cdot 1$ — separabel

$$\frac{1}{y-y^2} y' = 1$$

$$\int \frac{1}{y-y^2} \underbrace{y' \, dt}_{dy} = \int 1 \, dt$$

$$\int \frac{1}{y-y^2} \, dy = \int 1 \, dt$$

$$\int \left(\frac{1}{y} + \frac{1}{1-y}\right) dy = t + C$$

$$\ln|y| - \ln|1-y| = t + C$$

$$\int \frac{1}{y-y^2} \, dy = \cancel{\int (y-y^2)^{-1} \, dy}$$

$u = y - y^2$
$u' = 1 - 2y$

$$= \cancel{\int u^{-1} \frac{du}{1-2y}}$$

$$= \int \frac{1}{y \cdot (1-y)} \, dy$$

$$= \int \frac{A}{y} + \frac{B}{1-y} \, dy$$

$$1 = A \cdot (1-y) + B \cdot y$$

$$A = 1, \; B = 1$$

$$\ln|y| - \ln|1-y| = t + C$$

$$e^{\ln|y| - \ln|1-y|} = e^{t+C}$$

$$\frac{|y|}{|1-y|} = e^t \cdot e^C$$

$$\frac{y}{1-y} = \pm e^C \cdot e^t = Ke^t$$

$$y = Ke^t \cdot (1-y) = Ke^t - Ke^t \cdot y$$

$$y + Ke^t \cdot y = Ke^t$$

$$\parallel$$

$$y \cdot (1 + Ke^t)$$

$$y = \frac{Ke^t}{1+Ke^t}$$

② Lineære første ordens diff.likninger

(integrerende faktor)

<u>Defn:</u> En lineær første ordens diff.likn. har formen

$$y' + a(t)\cdot y = b(t)$$

der $a(t), b(t)$ er funksjoner i t.

<u>Enkelt tilfelle:</u> $a(t) = a$, $b(t) = b$ er konstanter

$$y' + ay = b \quad \Big| \cdot e^{at} \leftarrow \text{integrerende faktor}$$

$$\underbrace{y'e^{at} + ae^{at}y}_{=(y\cdot e^{at})'} = be^{at}$$

$$= (y\cdot e^{at})' = be^{at}$$

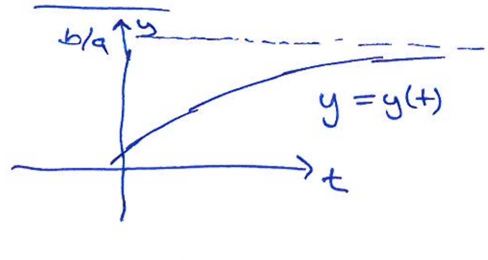
$$y\cdot e^{at} = \int be^{at}\,dt$$

$$y\cdot e^{at} = b\cdot\frac{1}{a}\cdot e^{at} + C$$

$$\underline{\underline{y = \frac{b}{a} + Ce^{-at}}} \leftarrow$$

generell formel når a, b er <u>konstanter</u>

Hvis $a>0$:



<u>Eks:</u>

$$\left.\begin{aligned} P' &= \lambda\cdot(D-S) \\ D &= a-bP \\ S &= \alpha+\beta P \end{aligned}\right\}\quad \lambda, a, b, \alpha, \beta \text{ er konstanter (positive)}$$

$$P' = \lambda\cdot\Big((a-bP)-(\alpha+\beta P)\Big)$$

$$P' = \lambda(a-\alpha) - \lambda(b+\beta)P$$

$$P' + \lambda(b+\beta)P = \lambda(a-\alpha)$$

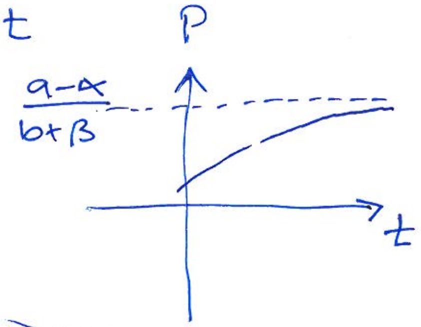
$$\Downarrow$$

$$P = \frac{\cancel{\lambda}(a-\alpha)}{\cancel{\lambda}(b+\beta)} + Ce^{-\lambda(b+\beta)t}$$

$$y' + ay = b$$

$$y = \frac{b}{a} + Ce^{-at}$$

$$\underline{P(t) = \frac{a-\alpha}{b+\beta} + C\cdot e^{-\lambda(b+\beta)t}}$$



<u>Med numeriske verdi (eksempel):</u>

$$\left.\begin{aligned} P' &= 0.5(D-S) \\ D &= 5000-4P \\ S &= 1000+6P \end{aligned}\right\}\quad \begin{aligned} &\lambda = 0.5 \\ &a = 5000, \; b = 4 \\ &\alpha = 1000, \; \beta = 6 \end{aligned}$$

$$\Downarrow$$

$$p(t) = \frac{4000}{10} + C\cdot e^{-0.5\cdot 10t}$$

$$= 400 + Ce^{-5t}$$

$$\left.\begin{aligned} p(0) &= 400 + C\cdot e^{0} \\ &= 400 + C \\ &\Downarrow \\ C &= p(0) - 400 \end{aligned}\right\} \Rightarrow \underline{\underline{p(t) = 400 + (p(0)-400)e^{-5t}}}$$

Lineære første ordens difflikninger – generelt & tilfelle

$$y' + a(t)\cdot y = b(t) \quad \Big|\cdot u = \text{integrerende faktor}$$

$$\underbrace{y'\cdot u + a(t)\cdot u\cdot y}_{} = b(t)\cdot u$$

$$(y\cdot u)' = b(t)\cdot u$$

$$y\cdot u = \int b(t)\cdot u\,dt$$

$$y = \frac{1}{u}\cdot\int b(t)\cdot u\,dt$$

Hvordan finne u
(integrerende faktor)

For at $(y\cdot u)' = y'u + a(t)u\,y$
så må vi ha

$$u' = a(t)\cdot u$$

Vi kan velge

$$u = e^{\int a(t)dt}$$

Ex:

$$y' + \overset{\overset{a(t)}{''}}{2t}\,y = \overset{\overset{b(t)}{''}}{t}$$

$$y'e^{t^2} + 2te^{t^2}\cdot y = te^{t^2}$$

$$(y\cdot e^{t^2})' = te^{t^2}$$

$$y\cdot e^{t^2} = \int te^{t^2}dt = \int \cancel{t}\cdot e^u\cdot\frac{du}{2\cancel{t}} = \frac{1}{2}e^u + C$$

$u = t^2$
$du = 2t\cdot dt$

$$= \frac{1}{2}\cdot e^{t^2} + C$$

Integrerende faktor:

$a(t) = 2t \Rightarrow \int 2t\,dt = t^2 + C$

$\Rightarrow u = e^{t^2}$

$$y\cdot e^{t^2} = \frac{1}{2}e^{t^2} + C$$

$$y = \frac{1}{2} + C\cdot e^{-t^2}$$

Eks: $y' + \underset{\substack{\parallel \\ a(t)}}{3t^2}\, y = e^{-t^3}$, $y(0) = 2$

$$u = e^{\int 3t^2 dt} = e^{t^3 + C} = e^{t^3}$$

$$\left(y \cdot e^{t^3}\right)' = e^{-t^3} \cdot e^{t^3} = 1$$

$$y \cdot e^{t^3} = \int 1 \, dt = t + C$$

$$y = \underline{\underline{t e^{-t^3} + C e^{-t^3}}} = \frac{t + C}{e^{t^3}}$$

$\underline{y(0) = 2}$: $\quad 2 = \dfrac{0 + C}{e^0} = C$

$$\underline{C = 2}$$

$$\Downarrow$$

$$y = \underline{\underline{\frac{t+2}{e^{t^3}}}}$$

Eks: $y' - \ln t \cdot y = t$

$$\left(y \cdot e^{t - t\ln t}\right)' = t e^{t - t\ln t}$$

$$y \cdot e^{t - t\ln t} = \int t e^{t - t\ln t} dt$$

(finner ikke løsning)

$a(t) = -\ln t$

$\int a(t) = \int -\ln t \, dt$

$= -(t \cdot \ln t - t) + C$

$= -t\ln t + t + C$

$u = e^{-t\ln t + t}$

$= \underline{e^t \cdot e^{-t\ln t}}$

$\left(= \dfrac{e^t}{\left(e^{\ln t}\right)^t} = \dfrac{e^t}{t^t}\right)$