FORELESNING 23

ELE 3719

EIVIND ERIKSEN

APR 19 2012

MATEMATIKK VF.

PLAN:

1. Eksamen 2010/11
2. Oppsummering ev.

Oppgave 6: $\min \int_0^{25} (8y^2 + 625\dot{y}^2) e^{-0.08t} \, dt$ når $\begin{cases} y(0) = 1 \\ y(25) = e^4 \end{cases}$

a) Euler-likningen: $F(t, y, \dot{y}) = (8y^2 + 625\dot{y}^2) e^{-0.08t}$

$$F'_y - \frac{d}{dt}(F'_{\dot{y}}) = 0$$

$$F'_y = 16y \cdot e^{-0.08t}$$

$$F'_{\dot{y}} = 1250\dot{y} \cdot e^{-0.08t}$$

$$\frac{d}{dt}(F'_{\dot{y}}) = \left(1250\dot{y}\, e^{-0.08t}\right)'_t = 1250\ddot{y}\, e^{-0.08t}$$

$$+ 1250\dot{y} \cdot e^{-0.08t} \cdot (-0.08)$$

$$= e^{-0.08t}(1250\ddot{y} - 100\dot{y})$$

Euler: $16y\, e^{-0.08t} - e^{-0.08t} \cdot (1250\ddot{y} - 100\dot{y}) = 0$

$$e^{-0.08t}(16y - 1250\ddot{y} + 100\dot{y}) = 0 \quad | : e^{-0.08t}$$

$$-1250\ddot{y} + 100\dot{y} + 16y = 0 \quad | : (-1250)$$

$$\ddot{y} - 0.08\dot{y} - 0.0128y = 0$$

b) Løs $\min \int_0^{25} (8y^2 + 625\dot{y}^2) e^{-0.08t} \, dt$ når $\begin{cases} y(0)=1 \\ y(25)=e^4 \end{cases}$

Kandidater for løsn: $\boxed{\text{Løsn. av Euler-likn + initialbetingelser.}}$

Euler: $\ddot{y} - 0.08\dot{y} - 0.0128y = 0$

Kar. likn.: $r^2 - 0.08r - 0.0128 = 0$

$$r = \frac{0.08 \pm \sqrt{0.08^2 + 0.0512}}{2}$$

$r_1 = 0.16 \qquad r_2 = -0.08$

Generell løsn: $y = C_1 \cdot e^{0.16t} + C_2 \cdot e^{-0.08t}$

$y(0)=1$:

$y(25)=e^4$

$1 = C_1 \cdot e^0 + C_2 e^0 \Rightarrow C_1 + C_2 = 1$

$C_2 = 1 - C_1$

$$e^4 = C_1 \cdot e^{0.16 \cdot 25} + (1-C_1) e^{-0.08 \cdot 25}$$

$$e^4 = C_1 \cdot e^4 + (1-C_1) \cdot e^{-2}$$

$$-e^{-2} + e^4 = C_1 \cdot (e^4 - e^{-2})$$

$$C_1 = \frac{e^4 - e^{-2}}{e^4 - e^{-2}} = 1 \qquad C_2 = 0$$

Konkl: Kandidat for min : $y = e^{0.16t}$

Er denne kandidaten <u>minimum</u>?

Sjekker om F er <u>konveks</u> som funksjon i $(y, \dot{y})$:

$$\left.\begin{aligned} F'_y &= 16y\, e^{-0.08t} \\ F'_{\dot{y}} &= 1250\dot{y}\, e^{-0.08t} \end{aligned}\right\} \quad \begin{aligned} F''_{yy} &= 16 \cdot e^{-0.08t} \\ F''_{y\dot{y}} &= 0 \\ F''_{\dot{y}\dot{y}} &= 1250\, e^{-0.08t} \end{aligned}$$

$$\boxed{\begin{aligned} &F''_{yy} \cdot F''_{\dot{y}\dot{y}} - (F''_{y\dot{y}})^2 \geq 0 \\ &F''_{yy},\ F''_{\dot{y}\dot{y}} \geq 0 \end{aligned}}$$

bet. for <u>konveks</u>

$16 \cdot 1250 \cdot \left(e^{-0.08t}\right)^2 \geq 0$ <u>ok</u>

$16\, e^{-0.08t},\ 1250\, e^{-0.08t} \geq 0$ <u>ok</u>

$\Downarrow$

F <u>konveks</u> i $(y, \dot{y})$

$\Downarrow$

$\underline{y = e^{0.16t}}$ er min.

<u>Minimal verdi:</u>

$$\int_0^{25} \Big[8 \underbrace{(e^{0.16t})^2}_{y} + 625 \cdot (\underbrace{0.16 \cdot e^{0.16t}}_{\dot{y}})^2 \Big] e^{-0.08t}\, dt$$

$$= \int_0^{25} \left(8\, e^{0.32t} + 625 \cdot 0.16^2 \cdot e^{0.32t} \right) e^{-0.08t}\, dt$$

$$= \int_0^{25} 8\, e^{0.24t} + 16\, e^{0.24t} = \int_0^{25} 24 \cdot e^{0.24t}\, dt$$

$$= \left[24 \cdot \frac{1}{0.24}\, e^{0.24t} \right]_0^{25} = \underline{100\left(e^6 - 1\right)} \simeq 40.243$$

Oppgave 5:

a) $y'' + 8y' + 16y = 32 \Rightarrow y = y_h + y_p$

y_h: $y'' + 8y' + 16y = 0$

Kar. Likning: $r^2 + 8r + 16 = 0$

$(r+4)^2 = 0 \qquad r = -4$

$$y_h = C_1 e^{-4t} + C_2 t e^{-4t}$$

y_p: $y'' + 8y' + 16y = 32$

Gjetter: $\begin{cases} y = A \\ y' = 0 \\ y'' = 0 \end{cases}$

$0 + 8 \cdot 0 + 16 \cdot A = 32$

$A = 2$

$y_p = 2$

Gen. løsn: $y = y_h + y_p = C_1 e^{-4t} + C_2 t e^{-4t} + 2$

b) $t^2 y' + 2t y = t e^{-t}$

lineær, løses vha
integrerende faktor

$$y' + \frac{2t}{t^2} \cdot y = \frac{t e^{-t}}{t^2}$$

$$y' + \frac{2}{t} \cdot y = \frac{e^{-t}}{t}$$

$$\boxed{y' + a(t) \cdot y = b(t)}$$

$$y' + \frac{2}{t} y = \frac{e^{-t}}{t} \quad | \cdot t^2$$

$$\underbrace{t^2 y' + 2t y}_{} = t e^{-t}$$

$$(t^2 \cdot y)' = t e^{-t}$$

$$t^2 y = \int t e^{-t} dt$$

$$y = \frac{1}{t^2} \int t e^{-t} dt$$

$$= \frac{-t e^{-t} - e^{-t} + C}{t^2}$$

$$= -\frac{e^{-t}}{t} - \frac{e^{-t}}{t^2} + \frac{C}{t^2}$$

Integrerende faktor:

$$e^{\int a(t) dt} = e^{\int \frac{2}{t} dt}$$

$$= e^{2 \ln |t|}$$

$$= e^{\ln |t|^2} = |t|^2 = t^2$$

$$\int \underset{v}{t} \, \underset{u'}{e^{-t}} dt$$

$u = -e^{-t}$ $v = t$
$u' = e^{-t}$ $v' = 1$

$$= uv - \int u v' dt$$

$$= -t e^{-t} - \int -e^{-t} \cdot 1 \, dt$$

$$= -t e^{-t} + \int e^{-t} dt$$

$$= -t e^{-t} - e^{-t} + C$$

Eksamen 06/2011:

Oppgave 3:

b) $\boxed{y' + 2e^t \cdot y = e^t}$, $y(0)=1$

$$y' = e^t - 2e^t y = e^t \cdot (1-2y)$$

$$\frac{1}{1-2y} y' = e^t \Rightarrow \int \frac{1}{1-2y}\, dy = \int e^t\, dt$$

$$\frac{1}{-2} \cdot \ln|1-2y| = e^t + C$$

$$\ln|1-2y| = -2e^t - 2C$$

$$|1-2y| = e^{-2e^t - 2C}$$

$$1-2y = \pm e^{-2e^t} \cdot e^{-2C}$$

$$1-2y = Ke^{-2e^t}$$

$$\frac{-2y}{-2} = \frac{Ke^{-2e^t} - 1}{-2}$$

$$\underline{y = \frac{1}{2}\left(1 - Ke^{-2e^t}\right)}$$

b) $y' + 2e^t y = e^t$

$a(t) = +2e^t$

$u = e^{\int a(t)\,dt}$

$= e^{+2e^t}$

$u = e^{2e^t}$

$y' \cdot e^{2e^t} + e^{2e^t} \cdot 2e^t \cdot y = e^{2e^t} \cdot e^t$

$(e^{2e^t} \cdot y)' = e^{2e^t} \cdot e^t$

$y \cdot e^{2e^t} = \int e^{2e^t} \cdot e^t\,dt$

$= \int e^v \cdot \frac{1}{2} \cdot dv$

$= \frac{1}{2} \int e^v dv$

$\frac{y \cdot \cancel{e^{2e^t}}}{\cancel{e^{2e^t}}} = \frac{1}{2} e^v + C = \frac{\frac{1}{2} e^{2e^t} + C}{e^{2e^t}}$

Substi

$v = 2e^t$

$v' = 2e^t$

$dv = 2e^t \cdot dt$

$y = \frac{1}{2} + C \cdot e^{-2e^t}$