FORELESNING 6 — ELE 3719

Eivind Eriksen — FEB 07 2012 — MATEMATIKK VALGFAG

PLAN: (32, 33/34, 37)

1. Oppgaveark 3 + Oppgaveark 2: (9, 11, 14)
2. Kontinuerlige fordelinger: Eksponensialfordeling, normalfordeling — [R] 2.3
3. (Forventning, varians – neste gang) — [R] 2.4

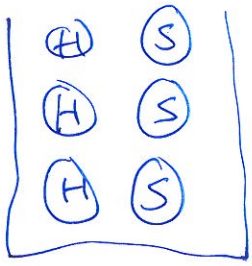
Oppgaveark 2:

9)

$$F(b) = \begin{cases} 0, & b < 0 \\ 1/2, & 0 \le b < 1 \\ 3/5, & 1 \le b < 2 \\ 4/5, & 2 \le b < 3 \\ 9/10, & 3 \le b < 3.5 \\ 1, & b \ge 3.5 \end{cases}$$

$$f(x) = \begin{cases} \frac{1}{2}, & x = 0 \\ \frac{1}{10}, & x = 1 \\ \frac{1}{5}, & x = 2 \\ \frac{1}{10}, & x = 3 \\ \frac{1}{10}, & x = 3.5 \end{cases}$$

11)



p (2 av de 4 første er hvite)

$= p(X=2)$

$= \binom{4}{2} \cdot \left(\frac{1}{2}\right)^2 \cdot \left(1-\frac{1}{2}\right)^2 = \frac{4 \cdot 3}{2} \cdot \frac{1}{16}$

$= \frac{6}{16} = \underline{\underline{3/8}}$

X = antall hvite på 4 gjentagelser.

binomisk : $n = 4$
$p = 3/6 = 1/2$

14) X binomial
$n = 6$, $p = 1/2$

$p(X=0) = \binom{6}{0}\left(\frac{1}{2}\right)^0 \cdot \left(\frac{1}{2}\right)^6 = 1 \cdot \left(\frac{1}{2}\right)^6$
$p(X=1) = \binom{6}{1}$ —"— $= 6 \cdot \left(\frac{1}{2}\right)^6$
$p(X=2) = \binom{6}{2}$ —"— $= 15 \cdot \left(\frac{1}{2}\right)^6$
$p(X=3) = \binom{6}{3}$ —"— $= 40 \cdot \left(\frac{1}{2}\right)^6$
$p(X=4) = \binom{6}{4}$ —"— $= 15 \cdot \left(\frac{1}{2}\right)^6$
$p(X=5) = \binom{6}{5}$ —"— $= 6 \cdot \left(\frac{1}{2}\right)^6$
$p(X=6) = \binom{6}{6}$ —"— $= 1 \cdot \left(\frac{1}{2}\right)^6$

4 a) – b)

	X = min	Y = maks
1 1	1	1
1 2	1	2
1 3		
1 4	⋮	⋮
1 5		
1 6	1	6
2 1		
2 2		
⋮		

Oppgaveark 3: 32, 33/34, 37

32) Kjøper lodd i 50 lotteri

Sann. vinne: $\frac{1}{100}$

X = antall vinnerlodd

$\boxed{\begin{array}{c}\text{binomisk}\\ n=50 \\ p=1/100\end{array}} \simeq \boxed{\begin{array}{c}\text{Poisson}\\ \lambda = np = 1/2\end{array}}$

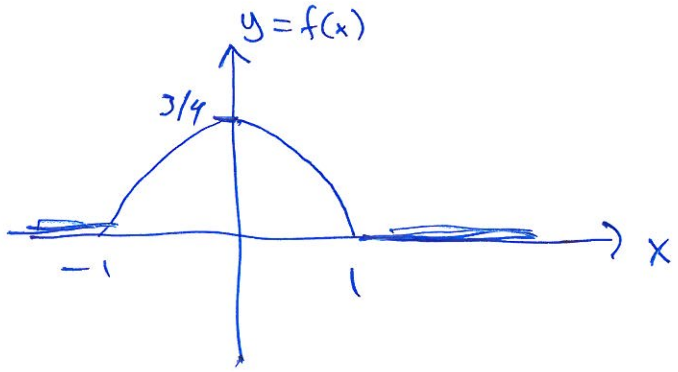
$P(X=i) = \frac{\lambda^i}{i!}e^{-\lambda}$

$$P(X\geq 1) \simeq 1 - P(X=0) = 1 - \frac{(1/2)^0}{0!}\cdot e^{-1/2} = \underline{0.39}$$

$$P(X=1) \simeq \frac{(1/2)^1}{1!} e^{-1/2} = \underline{0.30}$$

$$P(X\geq 2) = 1 - P(X=0) - P(X=1) \simeq \underline{0.09}$$

33). $f(x) = \begin{cases} c\cdot(1-x^2), & -1 < x < 1 \\ 0, & \text{ellers}\end{cases}$



a) Krav: i) $f(x)\geq 0$ for alle x $\iff$ $c\geq 0$

ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\Updownarrow$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-1}^{1} c(1-x^2)\,dx = c\cdot\left[x - \frac{1}{3}x^3\right]_{-1}^{1}$$

$$= c\cdot\left(\frac{2}{3} - \left(-1 - \frac{1}{3}(-1)^3\right)\right) = c\cdot\left(\frac{2}{3}+\frac{2}{3}\right)$$

$$= \frac{4}{3}c = 1 \implies c = \underline{\underline{\frac{3}{4}}}$$

b) $F(b) = p(X \le b) = \int_{-\infty}^{b} f(x)\,dx$

$\underline{-1 < b < 1:}$ $F(b) = \int_{-1}^{b} \frac{3}{4}\cdot(1-x^2)\,dx$

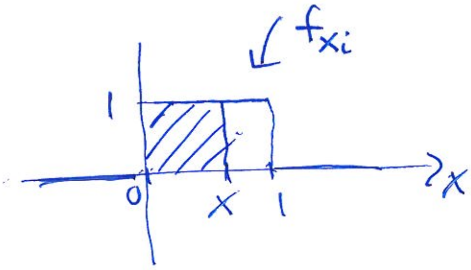
$= \frac{3}{4}\left[x - \frac{1}{3}x^3\right]_{-1}^{b} = \frac{3}{4}\cdot\left(\left(b - \frac{1}{3}b^3\right) - \left(-\frac{2}{3}\right)\right)$

$= \underline{\underline{\frac{3}{4}b - \frac{1}{4}b^3 + \frac{1}{2}}}$

$\left.\begin{array}{ll} b \le -1: & F(b) = 0 \\ b \ge 1: & F(b) = 1 \end{array}\right\}$ $F(b) = \begin{cases} 0, & b \le -1 \\ \frac{3}{4}b - \frac{1}{4}b^3 + \frac{1}{2}, & -1 < b < 1 \\ 1, & b \ge 1 \end{cases}$

37) $X_1, X_2, \ldots, X_n$ uavhengige var.

X_i : uniform på $\underline{(0,1)}$



$M = \max\{X_1, X_2, \ldots, X_n\}$

$$\boxed{F_M(x) = x^n \quad , \quad 0 < x < 1}$$

Anta $0 < x < 1$:

$F_M(x) = p(M \le x) = p(X_1 \le x \text{ og } X_2 \le x \text{ og } \ldots \text{ og } X_n \le x)$

$= p(X_1 \le x)\cdot p(X_2 \le x)\cdot \ldots \cdot p(X_n \le x) = \underline{\underline{x^n}}$

pga uavhengighet

$p(X_1 \le x) = \int_{-\infty}^{x} f_{X_1}(x)\,dx = \int_{0}^{x} 1\,dx = x$

Vet at $F_M(x) = x^n$ for $0 \leq x < 1$.

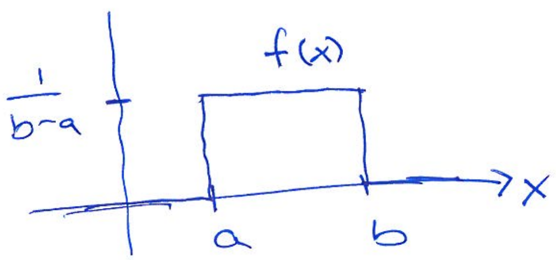
Hva er $f_M(x) = ?$

$$f_M(x) = F_M'(x) = \underline{\underline{n x^{n-1}}} \quad , \quad 0 < x < 1$$

② Kontinuerlige variabler

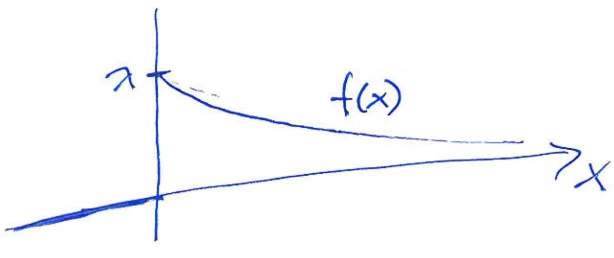
Standard kont. variabler

a) Uniform på $[a,b]$:



$$f(x) = \begin{cases} \frac{1}{b-a} & , \ a \leq x \leq b \\ 0 & , \ \text{ellers} \end{cases}$$

b) Eksponensialfordeling: (parameter $\lambda > 0$)



$$f(x) = \begin{cases} \lambda e^{-\lambda x} & , \ x \geq 0 \\ 0 & , \ \text{ellers} \end{cases}$$

Sjekk:

i) $f(x) \geq 0$ ok.

ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_0^{\infty} \lambda e^{-\lambda x}\,dx$$

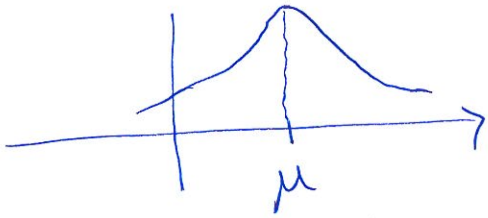
$$= \left[-e^{-\lambda x} \right]_0^{\infty}$$

$$= \underbrace{\lim_{x \to \infty} (-e^{-\lambda x})}_{=0} - (-e^0) = \underline{\underline{1}}$$

(Gammafordeling ikke pensum).

c) <u>Normalfordelingen</u> med parametre μ, σ^2

$$f(x) = \frac{1}{\sqrt{2\pi}\cdot\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

(dette kan bevises, men ikke mulig å regne ut vha "vanlig" integrasjon)

X normal (μ, σ) $\Rightarrow$ $Z = \frac{X-\mu}{\sigma}$

parametre

standard normalfordelt; dvs. normalfordelt med $\mu_Z = 0$, $\sigma_Z = 1$

$$f_Z(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

<u>Kumulativ fordelingsfunksjon for X:</u>

$$F_X(b) = P(X \le b) = P\left(\frac{X-\mu}{\sigma} \le \frac{b-\mu}{\sigma}\right)$$

$$= F_Z\left(\frac{b-\mu}{\sigma}\right)$$

Denne kan vi finne f.eks. ved hjelp av <u>tabeller</u> etc.
For å regne ut integralet, må vi bruke verktøy / tilnærminger; <u>ikke</u> mulig å regne ut vha "vanlig" integrasjon.