FORELESNING 9 ELE 3719

EIVIND ERIKSEN 16 FEB 2012 MATEMATIKK VALGFAG

PLAN:

1. Simultane fordelinger
   – kontinuerlig tilfelle
2. Dobbeltintegral

Simultane fordelinger – kontinuerlig tilfelle

Sannsynlighetstetthet: $f(x,y)$

diskret	kontinuerlig
$f(x,y) = p(x,y)$	$p(a \le X \le b,\ c \le Y \le d)$
$= p(X=x, Y=y)$	$= \int_a^b \int_c^d f(x,y)\, dy\, dx$

dobbelt-integral

Dobbeltintegral

$$\underbrace{\int_1^2\int_0^1 xy\,dy\,dx}_{\text{indre integral}} = \int_1^2 \frac{1}{2}x\,dx = \left[\frac{1}{2}\cdot\frac{1}{2}x^2\right]_1^2$$

$$= \frac{1}{4}\cdot\left(2^2-1^2\right) = \underline{\underline{\frac{3}{4}}}$$

Indre integral:

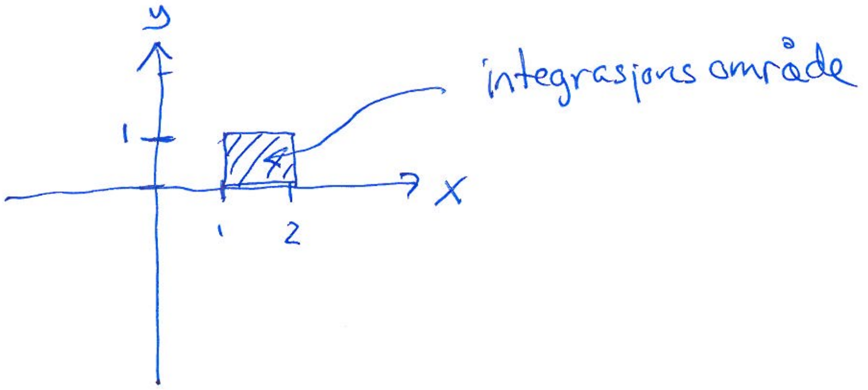
$$\int_0^1 xy\,dy = x\int_0^1 y\,dy = x\cdot\left[\frac{1}{2}y^2\right]_0^1$$

$$= x\left(\frac{1}{2}\cdot 1^2 - \frac{1}{2}\cdot 0^2\right) = \underline{\frac{1}{2}x}$$

x konstant siden integral i y

Tokning:

$$\int_1^2\int_0^1 xy\,dy\,dx = \frac{3}{4}$$



Regneregel:

$$\int_a^b \int_c^d f(x,y)\, dy\, dx = \int_c^d \int_a^b f(x,y)\, dx\, dy$$

Regn ut:

$$\int_0^1 \int_0^1 2x + e^y \, dy\, dx = \int_0^1 \Big[2xy + e^y \Big]_{y=0}^{y=1} dx$$

$$= \int_0^1 (2x + e) - 1 \; dx = \Big[x^2 + (e-1)x \Big]_0^1$$

$$= \Big(1^2 + 1\cdot(e-1)\Big) - \Big(0^2 + (e-1)\cdot 0\Big) = 1 + e - 1 = \underline{\underline{e}}$$

Vi ser at

$$f(x,y) = \begin{cases} \dfrac{2x+e^y}{e}, & 0 \le x,y \le 1 \\ 0, & \text{ellers} \end{cases}$$

er sannsynlighetstetthet.

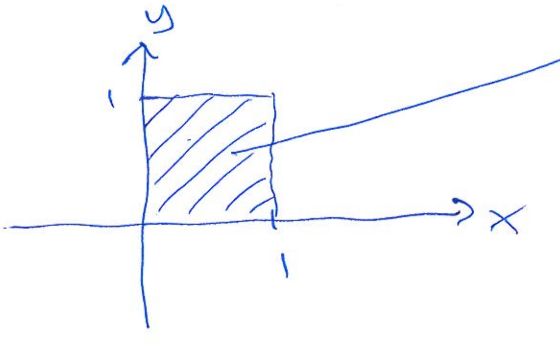
i) $f(x,y) \geq 0$

ii) $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)\, dy\, dx$

$$= \int_0^1 \int_0^1 \frac{2x+e^y}{e}\, dy\, dx$$

$$= \frac{1}{e} \cdot \int_0^1 \int_0^1 2x + e^y \, dy\, dx$$

$$= \frac{1}{e} \cdot e = \underline{\underline{1}}$$



Krav til sannsynlighets tetthet:

i) $f(x,y) \geq 0$ for alle x,y

ii) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, dy\, dx = 1$

Kumulativ fordelings funksjon:

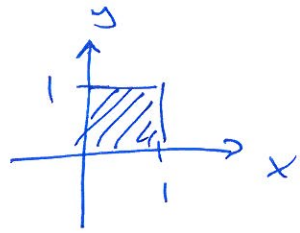
$$F(a,b) := P(X \leq a, Y \leq b) = \int_{-\infty}^{a}\int_{-\infty}^{b} f(x,y)\, dy\, dx$$

$$\left(\text{diskret}: \quad F(a,b) = P(X \leq a, Y \leq b) = \sum_{\substack{x \leq a \\ y \leq b}} f(x,y) \right)$$

Forventning:

$$E[g(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y) \cdot f(x,y)\, dy\, dx$$

Eks: $f(x,y) = \begin{cases} 1, & 0 \leq x,y \leq 1 \\ 0, & \text{ellers} \end{cases}$



(uniform på $[0,1] \times [0,1]$)

$$E(X) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x \cdot f(x,y)\, dy\, dx = \int_{0}^{1}\int_{0}^{1} x \cdot 1\, dy\, dx = \int_{0}^{1} [xy]_0^1\, dx$$

$$= \int_{0}^{1} x\, dx = \left[\tfrac{1}{2}x^2\right]_0^1 = \underline{\underline{\tfrac{1}{2}}}$$

$$E(XY) = \int_0^1 \int_0^1 xy \cdot 1 \, dy\, dx = \int_0^1 \left[x \cdot \frac{1}{2} y^2 \right]_{y=0}^{y=1} dx$$

$$= \int_0^1 \frac{1}{2} x \, dx = \left[\frac{1}{2} \cdot \frac{1}{2} x^2 \right]_0^1 = \underline{\underline{\frac{1}{4}}}$$

$$E(X) \cdot E(Y) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$E(X) = \mu_X$

<u>Defn:</u>

$$Var(X) = E\left((X - \mu_X)^2\right) = E(X^2) - E(X)^2$$

$$Var(Y) = E\left((Y - \mu_Y)^2\right) = E(Y^2) - E(Y)^2$$

$$Cov(X, Y) = E\left[(X - \mu_X) \cdot (Y - \mu_Y) \right]$$

$$= \underline{E(XY) - E(X) \cdot E(Y)}$$

<u>I Eks:</u>

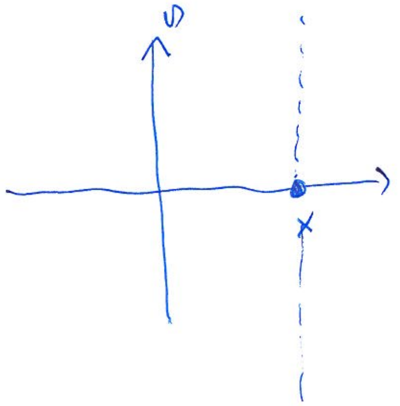
$$Cov(X, Y) = E(XY) - E(X) \cdot E(Y)$$

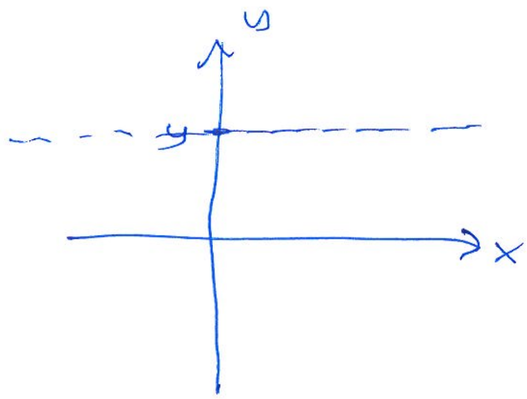
$$= 1/4 - 1/2 \cdot 1/2 = \underline{0}$$

$$f(x,y)$$

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy \qquad\qquad f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$$

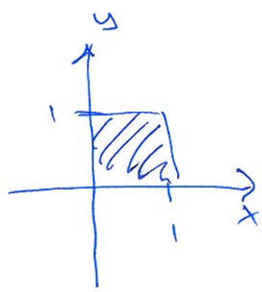
Sannsynlighetstetthet for X





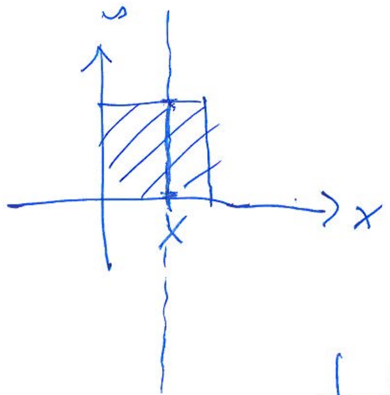
Eks:

$$f(x,y) = \begin{cases} \dfrac{2x+e^y}{e}, & 0 \le x,y \le 1 \\ 0, & \text{ellers} \end{cases}$$



$$E(X) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x \cdot f(x,y)\,dy\,dx = \int_0^1\int_0^1 x \cdot \frac{2x+e^y}{e}\,dy\,dx$$

Alt: $f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_0^1 \frac{2x+e^y}{e}\,dy$



$= \frac{1}{e}\Big[2xy + e^y\Big]_{y=0}^{y=1}$

$$f_X(x) = \begin{cases} \frac{1}{e}(2x+e-1) & , \quad 0 \le x \le 1 \\ 0 & , \quad \text{ellers} \end{cases}$$

$E(X) = \int_{-\infty}^{\infty} x \cdot f_X(x)\,dx = \cancel{\int_0^1 x \cdot \frac{2x+e^y}{e}\,dx}$

$= \int_0^1 x \cdot \frac{1}{e}(2x+e-1)\,dx$

$= \frac{1}{e}\int_0^1 2x^2 + x\cdot(e-1)\,dx$

$= \frac{1}{e}\Big[\frac{2}{3}x^3 + \frac{1}{2}x^2\cdot(e-1)\Big]_0^1$

$= \frac{1}{e}\left(\frac{2}{3} + \frac{1}{2}(e-1)\right) = \underline{\underline{\frac{2}{3e} + \frac{e-1}{2e}}}$

Viktig formel:

$$\frac{\partial^2}{\partial a \partial b} F(a,b) = f(a,b)$$

eller

$$\boxed{F''_{xy}(x,y) = f(x,y)}$$