Plan

1 Introduksjon til kurset
2 Vektorer og vektorregning
3 Indreprodukt og lengden til vektorer

Kontortid: B4-032
Onsdag kl 12-14

Temaer:
* Vektorer og matriser
* Sannsynlighetsregning
* Diff.likninger / variasjonsregning

[DA] Digital arbeidsbok.

2 Vektorer og vektorregning

Defn: En kolonnevektor er en matrise som består av en kolonne.

Eks: $\underline{v} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$ kolonnevektor, 3-vektor

$\underline{v} = \vec{v}$

Regneoperasjoner: i) Addisjon: $\underline{v} + \underline{w}$

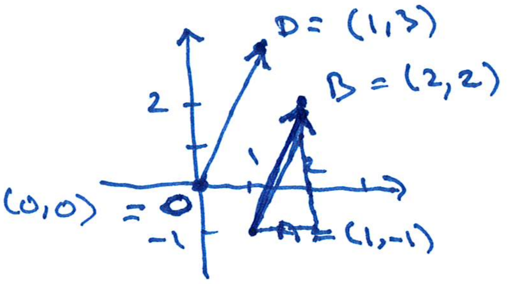
Eks: $\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 4 \end{pmatrix}$

ii) Skalarmultiplikasjon: $r \cdot \underline{v}$ (r = et tall)

Eks: $3 \cdot \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ 12 \end{pmatrix}$

Nullvektor: $\underline{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

Vektorer og koordinater:

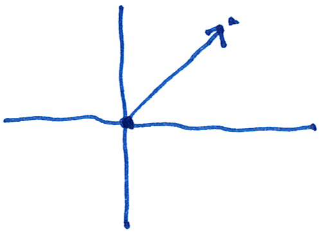


$$\vec{AB} = (1,3) = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\vec{OD} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

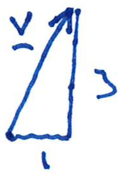
$$\left.\begin{array}{l} A = (x_1, y_1) \\ B = (x_2, y_2) \end{array}\right\} \quad \vec{AB} = (x_2 - x_1, y_2 - y_1) = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix}$$

Skrivemåte: $\mathbb{R}^2$ = mengden av alle 2-vektorer



Lengden av en vektor: $\|\underline{v}\|$

$n=2$: $\underline{v} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

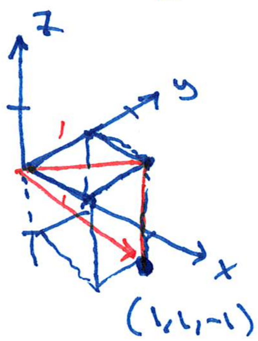


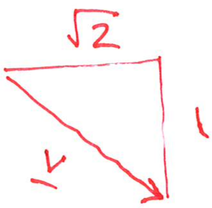
$$\|\underline{v}\| = \sqrt{1^2 + 3^2} = \underline{\underline{\sqrt{10}}}$$

$$\underline{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix} \qquad \|\underline{v}\| = \sqrt{v_x^2 + v_y^2}$$

$n=3$:

$\underline{v} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$





$$\|\underline{v}\| = \sqrt{(\sqrt{2})^2 + 1^2} = \underline{\underline{\sqrt{3}}}$$

$$\underline{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} : \quad \|\underline{v}\| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

Eivind Eriksen, Kontor B4-032, eivind.eriksen@bi.no
http://www.dr-eriksen.no/teaching/ELE3719/

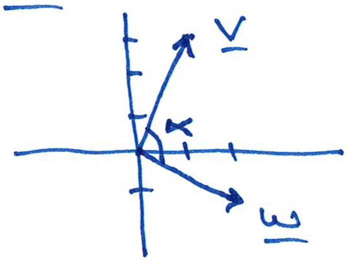
3 Indreprodukt: (prikkeprodukt)

Eks: $\underline{v} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ $\underline{w} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ $\underline{v} \cdot \underline{w} = 1 \cdot 2 + 3 \cdot (-1) = 2 - 3 = \underline{-1}$

Generell defn:

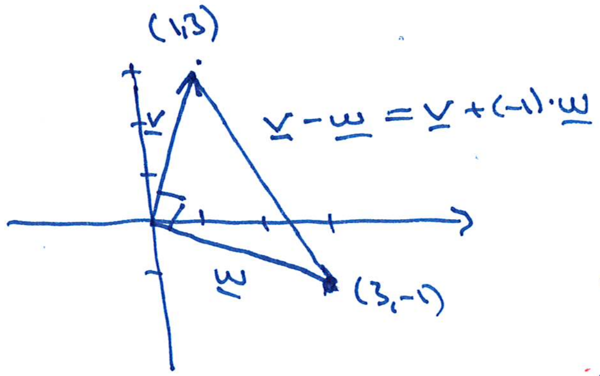
$\underline{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}$, $\underline{w} = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix}$: $\underline{v} \cdot \underline{w} = v_1 w_1 + v_2 w_2 + \ldots + v_n \cdot w_n$

Eks:



$$\boxed{\begin{array}{lll} \underline{v} \cdot \underline{w} = 0 & \Longleftrightarrow & \alpha = 90^\circ \\ \underline{v} \cdot \underline{w} > 0 & \Longleftrightarrow & \alpha < 90^\circ \\ \underline{v} \cdot \underline{w} < 0 & \Longleftrightarrow & \alpha > 90^\circ \end{array}}$$

$\underline{v} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ $\underline{w} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ $\therefore$ $\underline{v} \cdot \underline{w} = 1 \cdot 3 + 3 \cdot (-1) = 0$



$\|\underline{v}\| = \sqrt{1^2 + 3^2} = \sqrt{10}$

$\|\underline{w}\| = \sqrt{3^2 + (-1)^2} = \sqrt{10}$

$\underline{v} - \underline{w} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \underline{\begin{pmatrix} -2 \\ 4 \end{pmatrix}}$

$\|\underline{v} - \underline{w}\| = \sqrt{(-2)^2 + 4^2} = \sqrt{20}$

$\|v\|^2 + \|\underline{w}\|^2 = \|\underline{v} - \underline{w}\|^2$

Defn: To n-vektorer $\underline{v}, \underline{w}$ kalles ortogonale hvis $\underline{v} \cdot \underline{w} = 0$. Vi skriver i så fall at $\underline{v} \perp \underline{w}$.

Egenskaper til indreprodukt:

i) $\underline{v} \cdot \underline{w} = \underline{w} \cdot \underline{v}$

ii) $\underline{v} \cdot (\underline{w_1} + \underline{w_2}) = \underline{v} \cdot \underline{w_1} + \underline{v} \cdot \underline{w_2}$

iii) $\underline{v} \cdot (c \cdot \underline{w}) = c \cdot (\underline{v} \cdot \underline{w})$

iv) $\underline{v} \cdot \underline{v} = v_1^2 + v_2^2 + \dots + v_n^2 = \|\underline{v}\|^2 \geq 0$ $\left(\underline{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} \right)$

Oppsummering: $\underline{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}$ $\underline{w} = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{pmatrix}$

1) Addisjon: $\underline{v} + \underline{w} = \begin{pmatrix} v_1 + w_1 \\ v_2 + w_2 \\ \vdots \\ v_n + w_n \end{pmatrix}$

2) Skalarmultiplikasjon: $c \cdot \underline{v} = \begin{pmatrix} c v_1 \\ c v_2 \\ \vdots \\ c v_n \end{pmatrix}$

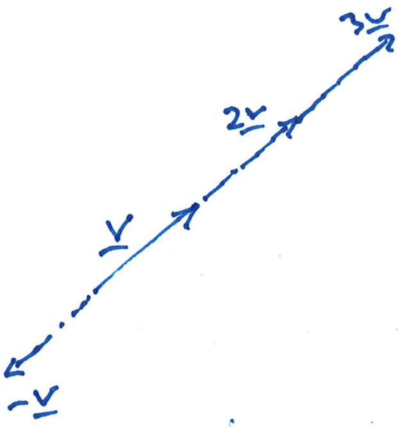
3) Lengden til en vektor: $\|\underline{v}\| = \sqrt{v_1^2 + v_2^2 + \dots + v_n^2}$

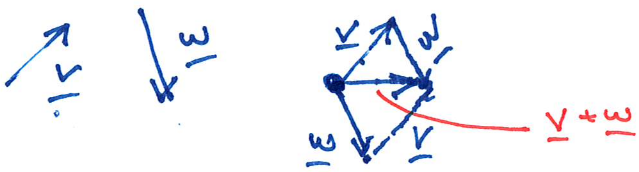
$\|\underline{v}\| \geq 0$, og $\|\underline{v}\| = 0$ bare hvis $\underline{v} = \underline{0}$

4) Indreprodukt: $\underline{v} \cdot \underline{w} = v_1 \cdot w_1 + v_2 \cdot w_2 + \dots + v_n \cdot w_n$.

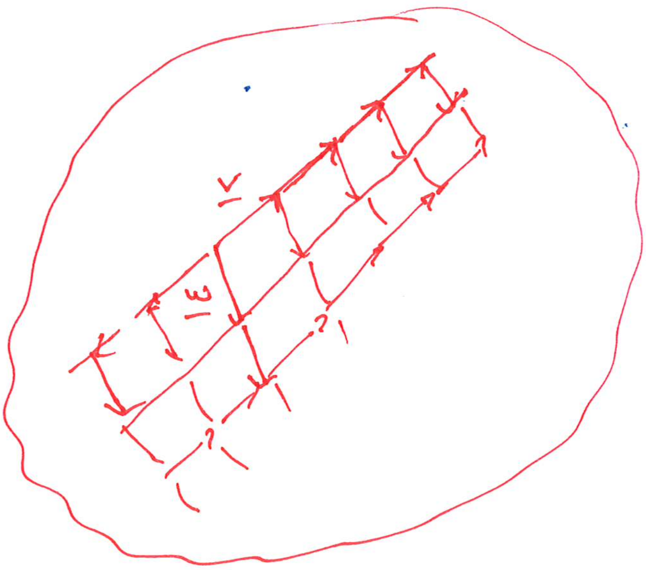
Husk: $\underline{v}, \underline{w}$ er ortogonale $\iff \underline{v} \cdot \underline{w} = 0$

Geometrisk tolkning av addisjon / skalar multiplikasjon





→ En lineærkombinasjon av $\underline{v}$ og $\underline{w}$: $c \cdot \underline{v} + d \cdot \underline{w}$ (c, d er tall)



Resultat: Cauchy-Schwartz ulikhet

For alle n-vektorer $\underline{v}, \underline{w}$ har vi at

$$|\underline{v} \cdot \underline{w}| \leq \|\underline{v}\| \cdot \|\underline{w}\|$$

$$\Updownarrow$$

$$-1 \leq \frac{\underline{v} \cdot \underline{w}}{\|\underline{v}\| \cdot \|\underline{w}\|} \leq 1$$

$$\Updownarrow$$

$$\boxed{\underline{v} \cdot \underline{w} = \|\underline{v}\| \cdot \|\underline{w}\| \cdot \cos(\alpha)}$$