Plan

1 Median og prosentiler
2 Dobbeltintegraler

Repetisjon: X kont. stokastisk variabel med verdier x

$f(x) = f_X(x)$ sannsynlighets-tettheten til X

$F(x) = F_X(x)$ fordelingsfunksjonen til X

Krav:
i) $f(x) \geq 0$
ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$P(a \leq X \leq b) = \int_a^b f(x)\,dx$

Krav:
i) F voksende funksjon
ii) $\lim_{x \to -\infty} F(x) = 0$, $\lim_{x \to \infty} F(x) = 1$

$P(X \leq x) = F(x)$

$f(x) \longrightarrow F(x) = \int_{-\infty}^{x} f(x)\,dx$

$f(x) = F'(x) \longleftarrow F(x)$

Forventning og Varians:

$E[X] = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$

forventingsverdien til X

$Y = h(X)$: $E(Y) = \int_{-\infty}^{\infty} h(x) \cdot f_X(x)\,dx$
$= \left(\int_{-\infty}^{\infty} y \cdot f_Y(y)\,dy \right)$

$\mathrm{Var}(X) = E[(X-\mu)^2]$

hvor $\mu = E(X)$

$= \int_{-\infty}^{\infty} (x-\mu)^2 \cdot f_X(x)\,dx$

i) $\mathrm{Var}(X) \geq 0$
ii) $\mathrm{Var}(X) = E(X^2) - E(X)^2$
$= \int_{-\infty}^{\infty} x^2 f(x)\,dx - \left(\int_{-\infty}^{\infty} x f(x)\,dx\right)^2$

Regneregler: i) $E(aX+b) = a\cdot E(X) + b$

$$\int (ax+b) f(x)\,dx = \int a x f(x)\,dx + \int b f(x)\,dx$$

$$= a\cdot \underbrace{\int x f(x)\,dx}_{E(X)} + b \underbrace{\int f(x)\,dx}_{=1} = aE(X) + b$$

Bruker at $Var(X) = E(X^2) - E(X)^2$

ii) $Var(aX+b) = E[(aX+b)^2] - (E[aX+b])^2$

$$= E[a^2X^2 + 2abX + b^2] - (a\cdot E(X) + b)^2$$

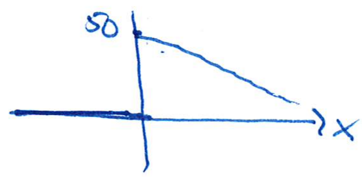
$$= E(a^2X^2) + E(2abX) + E(b^2) - \left(a^2E(X)^2 + 2abE(X) + b^2\right)$$

$$= a^2E(X^2) + \cancel{2abE(X)} + \cancel{b^2} - a^2E(X)^2 - \cancel{2abE(X)} - \cancel{b^2}$$

$$= a^2\left(E(X^2) - E(X)^2\right) = a^2\cdot Var(X)$$

Altså: $Var(aX+b) = a^2 \cdot Var(X)$

Ex: $f(x) = \begin{cases} k\, e^{-0.02x} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$



Er dette en sann. tetthet for en X?

i) $f(x) \geq 0$ for alle x (ok)

ii) $\int_{-\infty}^{\infty} f(x)\,dx = \int_0^{\infty} k e^{-0.02x}\,dx = \left[k\left(-\frac{1}{0.02}\right)e^{-0.02x}\right]_0^{\infty}$

$$= \left[-\frac{k}{0.02} e^{-0.02x}\right]_0^{\infty} = -\frac{k}{0.02}(0-1) = \frac{k}{0.02}$$

$$\frac{k}{0.02} = 1 \Rightarrow k = 0.02$$

Konklusjon: $f(x) = \begin{cases} 0.02\, e^{-0.02x} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$ er en tetthetsfn.

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$\underline{X}$:
$$f(x) = \begin{cases} 0.02\, e^{-0.02x} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$$

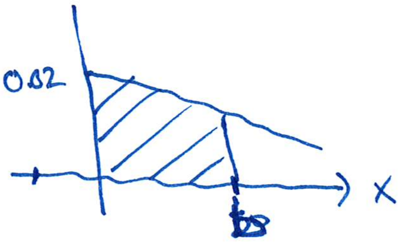
$$E(X) = \int_0^\infty \underline{x} \cdot \underline{0.02 e^{-0.02x}}\, dx = \int_0^\infty u'v\, dx$$

$$\boxed{\begin{array}{ll} u = -e^{-0.02x} & v = x \\ u' = 0.02 e^{-0.02x} & v' = 1 \end{array}}$$

$$= [uv]_0^\infty - \int_0^\infty uv'\, dx = -x e^{-0.02x} - \int -e^{-0.02x}\, dx$$

$$= \left[-x e^{-0.02x} - \frac{1}{0.02} e^{-0.02x} \right]_0^\infty$$

$$= \Big(\underbrace{\lim_{x\to\infty} \frac{-x}{e^{0.02x}}}_{0} - 0 \Big) - \Big(-0 \cdot e^{-0.02 \cdot 0} - \frac{1}{0.02} \Big)$$



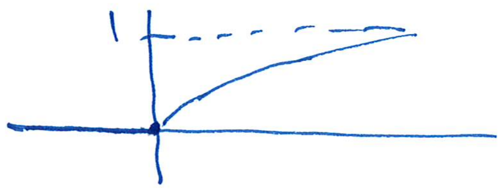
$$= \frac{1}{0.02} = \underline{\underline{50}}$$

$$F(b) = P(X \leq b) = \int_0^b f(x)\, dx = \int_0^b 0.02\, e^{-0.02x}\, dx$$

$$= \Big[-e^{-0.02x} \Big]_0^b = \left(-e^{-0.02b}\right) - \left(-e^0\right)$$

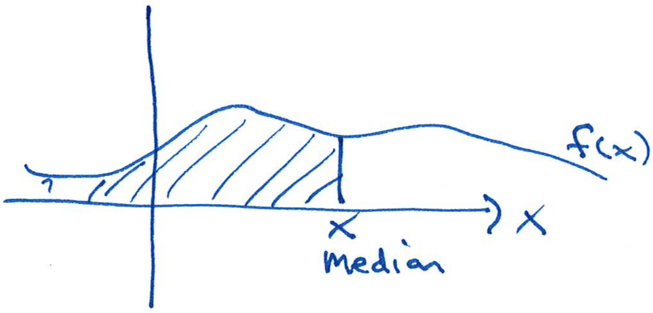
$$= \underline{1 - e^{-0.02b}}, \quad b \geq 0$$

$$F(x) = \begin{cases} 1 - e^{-0.02x} & , x \geq 0 \\ 0 & x < 0 \end{cases}$$



$$P(X \leq 50) = 1 - e^{-0.02 \cdot 50}$$
$$= 1 - e^{-1} = \underline{\underline{1 - 1/e}}$$

① Median og prosentiler



x median: $P(X \le x) = 0.50$

$$\Updownarrow$$

$$\int_{-\infty}^{x} f(x)\,dx = 0.50$$

$$\Updownarrow$$

$$F(x) = 0.50$$

Eks: X: $f(x) = \begin{cases} 0.02 \cdot e^{-0.02x} & , x \ge 0 \\ 0 & , x < 0 \end{cases}$

$$F(x) = 1 - e^{-0.02x} \quad , x \ge 0$$

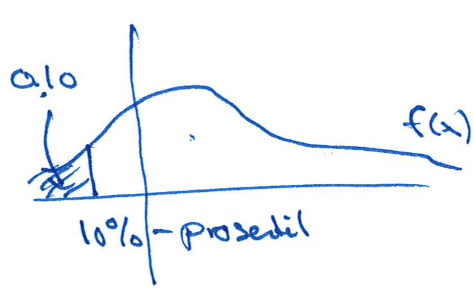
Median: $F(x) = 1 - e^{-0.02x} = 0.5$

$$1 - 0.5 = e^{-0.02x}$$

$$e^{-0.02x} = 0.5 \quad | \ln$$

$$-0.02x = \ln(1/2) = -\ln 2$$

$$x = \frac{-\ln 2}{-0.02} = \underline{50 \ln 2} \simeq \underline{\underline{32}}$$



p % - prosentil : x

$$P(X \le x) = p/100$$

10'ende prosentil:

$$F(x) = 0.10$$

$$1 - e^{-0.02x} = 0.10$$

$$e^{-0.02x} = 1 - 0.10 = 0.90$$

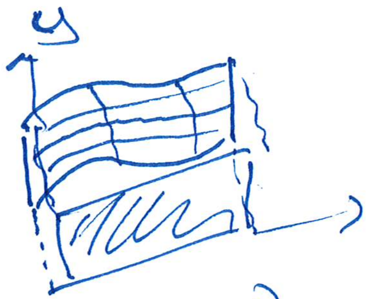
$$-0.02x = \ln(0.90)$$

$$x = \frac{\ln(9/10)}{-0.02}$$

$$= \frac{-\ln(10/9)}{-0.02}$$

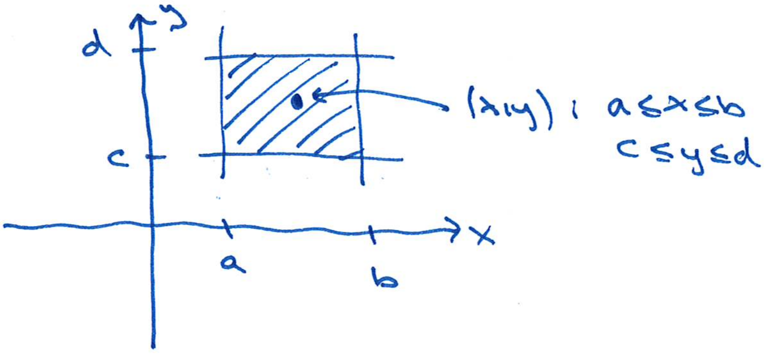
$$= \underline{50 \cdot \ln(10/9)}$$

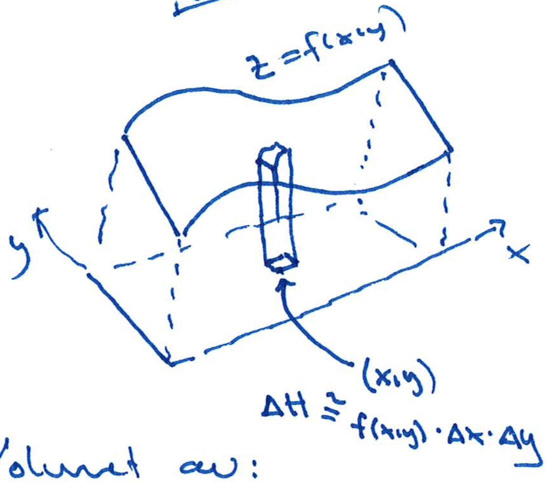
2 Dobbeltintegral



Anta $f(x,y) \ge 0$ for

$a \le x \le b$, $c \le y \le d$.





$$\int_{x=a}^{x=b}\int_{y=c}^{y=d} f(x,y)\,dy\,dx$$

Volumet av:

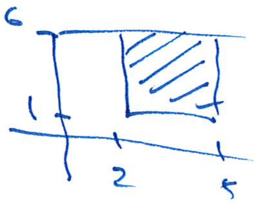
$a \le x \le b$

$c \le y \le d$

$0 \le z \le f(x,y)$

Eks: $f(x,y) = 4xy + 3y + 1$

når $2 \le x \le 5$, $1 \le y \le 6$



$$\int_2^5 \underbrace{\int_1^6 4xy + 3y + 1 \,dy}\,dx$$

$$= \int_2^5 \left[x \cdot 2y^2 + 3 \cdot \tfrac{1}{2} y^2 + y \right]_1^6 \,dx$$

$$= \int_2^5 \left(x \cdot 2 \cdot 6^2 + \tfrac{3}{2} \cdot 6^2 + 6\right) - \left(x \cdot 2 \cdot 1^2 + \tfrac{3}{2} \cdot 1^2 + 1\right) dx$$

$$= \int_2^5 72x + 54 + 6 - 2x - 3/2 - 1 \,dx = \int_2^5 70x + 57.5 \,dx$$

$$= \left[70 \cdot \frac{1}{2}x^2 + \frac{115}{2}x\right]_2^5 = \left(\frac{70}{2} \cdot 5^2 + \frac{115}{2} \cdot 5\right) - (140 + 115)$$

$$= \frac{70 \cdot 25 + 115 \cdot 5}{2} - 255 = 1162.5 - 255 = \underline{\underline{907.5}}$$

det var jo helt riktig!