	Plan
1	Lineære første ordens differensial-likninger
2	Integrerende faktor

Rep: Separable diff.likninger

$$y' = f(y) \cdot g(t) \quad | : f(y)$$

$$\frac{1}{f(y)} y' = g(t) \quad | \int \dots dt$$

$$\int \frac{1}{f(y)} dy = \int g(t)\, dt \qquad dy = y'dt$$

Eks: $\dfrac{(1+t^3)y'}{1+t^3} = \dfrac{t^2 y}{1+t^3}$

$$y' = \frac{t^2 \cdot y}{1+t^3} = y \cdot \frac{t^2}{1+t^3}$$

$$\frac{1}{y} y' = \frac{t^2}{1+t^3}$$

$$\int \frac{1}{y} dy = \int \frac{t^2}{1+t^3} dt \qquad \boxed{\begin{aligned} u &= 1+t^3 \\ du &= 3t^2 \cdot dt \end{aligned}}$$

$$\ln|y| = \int \frac{\cancel{t^2}}{u} \frac{1}{3\cancel{t^2}} du = \frac{1}{3} \int \frac{1}{u} du$$

$$\ln|y| = \frac{1}{3} \ln|u| + C = \frac{1}{3} \ln|1+t^3| + C$$

$$|y| = e^{\frac{1}{3}\ln|1+t^3| + C} = e^C \cdot e^{\ln |1+t^3|^{1/3}}$$

$$|y| = e^C \cdot |1+t^3|^{1/3}$$

$y = K\,(1+t^3)^{1/3}$ $\quad (K = \pm e^C)$

$$\underline{\underline{y = K \cdot \sqrt[3]{1+t^3}}}$$

① Lineære første ordens diff. likninger

Defn: En første ordens diff. likning kalles lineær hvis den kan skrives

$$\boxed{y' + a(t)\cdot y = b(t)} \iff y' = b(t) - a(t)y$$

Første ordens diff. likn: $y' = F(t,y)$

homogen : $b(t) = 0$ (ellers: inhomogen)

konstante koeff.: $a(t) = a$ er en konstant

Kalles lineær fordi venstresiden er lineær i y, y'

Eks:

$y' = 2y \iff y' - 2y = 0$ lineær, homogen, konstante koeff.

$2y' - 4y = 12 \iff y' - 2y = 6$
$y' = y + t \iff y' - y = t$
lineær, inhomogen, konstante koeff.

$(1+t^3)y' = t^2 \cdot y \iff y' = \dfrac{t^2 y}{1+t^3}$

$\Updownarrow$

$y' - \dfrac{t^2}{1+t^3}\, y = 0$

lineær, homogen, ikke konstante koeff.

$y' = y^2 \cdot t$ ikke lineær

(2) Metode: Integrerende faktor

$$y' + a(t)y = b(t)$$

Kan brukes for alle lineære første ordens diff.likn.

Eks: $y' - 2y = 6 \quad | \cdot u(t)$

$$\int (y' - 2y)\,dt = \int 6\,dt$$

$$\underbrace{y' \cdot u - 2y \cdot u}_{(y \cdot u)' = y' u + y \cdot u'} = 6 \cdot u$$

(1) Skal velge integrerende faktor u slik at

$$-2yu = y \cdot u'$$

$$\Updownarrow$$

$$-2u = u'$$

$$-2 = \frac{1}{u} u'$$

$$\int -2\,dt = \int \frac{1}{u}\,du$$

$$-2t + C = \ln|u|$$

$$|u| = e^{-2t+C}$$

$$u = K \cdot e^{-2t}$$

Velger $u = e^{-2t}$

$$(y \cdot e^{-2t})' = 6e^{-2t}$$

$$\int (y \cdot e^{-2t})'\,dt = \int 6e^{-2t}\,dt$$

$$y \cdot e^{-2t} = \int 6e^{-2t}\,dt$$

$$\boxed{\begin{matrix} u = -2t \\ du = -2dt \end{matrix}}$$

$$= \int 6e^{u} \cdot \frac{1}{(-2)}\,du$$

$$= -3e^{u} + C$$

$$ye^{-2t} = -3e^{-2t} + C \quad | \cdot e^{2t}$$

$$y = \underbrace{-3}_{y_p} + \underbrace{C \cdot e^{2t}}_{y_h}$$

Eks: $y' - 2ty = 6t \quad | \cdot u$

$$\underbrace{y' \cdot u - 2ty \cdot u}_{(y\cdot u)' \,=\, y'\cdot u + y \cdot u'} = 6t \cdot u$$

Trenger:

$-2ty \cdot u = y \cdot u'$

$\Updownarrow$

$-2tu = u'$

$\frac{1}{u} \cdot u' = -2t$

$\ln|u| = \int -2t\,dt$

$\ln|u| = -t^2 + C$

$|u| = e^{-t^2+C}$

$u = K \cdot e^{-t^2}$

Velger: $u = e^{-t^2}$

$$(y \cdot e^{-t^2})' = (y \cdot u)' = 6t \cdot u = 6t \cdot e^{-t^2}$$

$$y \cdot e^{-t^2} = y \cdot u = \int 6tu\,dt = \int 6te^{-t^2}\,dt$$

$$y = \frac{1}{u(t)} \int 6t \cdot u(t)\,dt$$

$\boxed{\begin{matrix} u = -t^2 \\ du = -2t\,dt \end{matrix}}$

$$= \int 6te^{u} \cdot \frac{1}{(-2t)}\,du$$

$$= -3e^{u} + C$$

$$= -3e^{-t^2} + C$$

$$y \cdot e^{-t^2} = -3e^{-t^2} + C \quad | \cdot e^{t^2}$$

$$y = -3 + Ce^{t^2}$$

Generell metode:

$y' + a(t)y = b(t)$ gir integrerende faktor $u = e^{\int a(t)\,dt}$

Generell løsning:

$$y = \frac{1}{u(t)} \cdot \int b(t) \cdot u(t)\,dt$$

Eks: $(t^2+1)y' + 2t\cdot y = 1$, $y(0)=2020$

$(t^2+1)\cdot \;|\quad y' + \frac{2t}{t^2+1}\cdot y = \frac{1}{t^2+1}$

$(t^2+1)\cdot y' + 2t y = 1$

$\big((t^2+1)\cdot y\big)' = 1$

$(t^2+1)\, y = t + C$

$y = \dfrac{t+C}{t^2+1}$

lineær $a(t) = \frac{2t}{t^2+1}$ $b(t) = \frac{1}{t^2+1}$

integrerende faktor:

$\int a(t)\,dt = \int \frac{2t}{t^2+1}\,dt$

$= \ln(t^2+1) + C$

$\Downarrow$

$u = e^{\ln(t^2+1)} = t^2+1$

$y(0)=2020$: $2020 = \dfrac{C}{1}$

$C = 2020$

Løsn. $y = \dfrac{t+2020}{t^2+1}$

Eks: $(1+t^3)y' = t^2 y$

$(1+t^3)y' - t^2 y = 0$

lineær, homogen

$y' - \frac{t^2}{1+t^3}y = 0$

$(y\cdot u)' = 0$

$y\cdot u = C$

$y = \frac{C}{u}$

$= C\cdot(1+t^3)^{1/3}$

$= C\cdot\sqrt[3]{1+t^3}$

Int. faktor:

$e^{-1/3\cdot\ln|1+t^3|+C} =$

$e^C\cdot e^{\ln|1+t^3|^{-1/3}} =$

$\pm e^C\cdot(1+t^3)^{-1/3}$

$\Rightarrow u = (1+t^3)^{-1/3}$

$\int -\frac{t^2}{1+t^3}\,dt =$ $\boxed{\begin{matrix} u = 1+t^3 \\ du = 3t^2\,dt\end{matrix}}$

$= \int \frac{-\cancel{t^2}}{u}\,\frac{1}{3\cancel{t^2}}\,du$

$= -\frac{1}{3}\ln|u| + C =$

$= -\frac{1}{3}\ln|1+t^3| + C$

Alt. metode:

$$y' + a(t)\cdot y = b(t)$$

lineær, første ordens diff. likning

$$\Downarrow$$

Superposisjons-prinsippet:

Generell løsning:

$y = y_h + y_p$ der

y_h: generell løsn. av $\boxed{y' + a(t)\cdot y = 0}$ ← alle løsn.

y_p: partikulær løsn. av $\boxed{y' + a(t)y = b(t)}$ ← én løsn. (spesialtilfelle)

Eks: $y' + ay = b$ (a, b er konst.)

$$y = y_h + y_p = \underline{C\cdot e^{-at} + \frac{b}{a}}$$

y_h: $y' + ay = 0$

$y' = -ay$

$y_h = C\cdot e^{-at}$

$\frac{1}{y}\cdot y' = -a$

$\int \frac{1}{y}\,dy = \int -a\,dt$

$\ln|y| = -at + C$

$|y| = e^{-at+C}$

$y = \pm e^{C}\cdot e^{-at}$

$y = K\cdot e^{-at}$

y_p: $y' + ay = b$

Konstant løsn: $y_p = A = \frac{b}{a}$

$y' = 0$

$0 + a\cdot A = b$

$A = b/a$