

4.2.1 En enkel bit (0 eller 1) kan overkøres F (feil) eller riktig (R). Det er gitt at

$$i) \quad p(F) = p \quad \Rightarrow \quad p(R) = 1 - p$$

ii) feil i ulike "bits" er uavh. av hverandre

Ved oversending av  $n$  bits, er antall feil  $X$  diforbinomisk fordelt (parametre  $p, n$ ).

a) Minst én feil:

$$p(X \geq 1) = 1 - p(X < 1) = 1 - p(X=0)$$

$$= 1 - \binom{n}{0} \cdot p^0 \cdot (1-p)^n = 1 - (1-p)^n$$

$$\stackrel{\uparrow}{p(X < 1)} = p(X=0) : \quad p(\underbrace{RRR \dots R}_{n \text{ ganger}}) = (1-p) \cdot (1-p) \cdots (1-p) = (1-p)^n$$

b) Feil signal = ~~minst~~<sup>minst</sup> to feil av tre:  $X$  binomial  $(p, n=3)$

$$p(X \geq 2) = p(X=2) + p(X=3)$$

$$= \binom{3}{2} p^2 \cdot (1-p) + \binom{3}{3} p^3$$

$$= \underline{3p^2(1-p) + p^3} = 3p^2 - 2p^3$$

4.2.2:

F: flymoter svikter  $p(F) = q$

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n flymoter, uavh.:  $X$  = antall motorer som svikter

binomial (q, n)

a) Minst to svikter i like,  $n=4$ : (Målt to svikter)

$$P(X \leq 2) = 1 - p(x=3) - p(x=4)$$

$$= 1 - \underline{4q^3(1-q)} - q^4$$

Kan også skrives:

$$\begin{aligned} P(X \leq 2) &= p(x=0) + p(x=1) + p(x=2) \\ &= \underline{(1-q)^4 + 4q(1-q)^3 + 6q^2(1-q)^2} \end{aligned}$$

disse uttrykkene er like

b) Mint to svikter i like,  $n=3$  (målt 1 svikter)

$$P(X \leq 1) = 1 - p(x=2) - p(x=3)$$

$$= 1 - 3q^2(1-q) - q^3$$

Sannsynligg. sammenligning: Med  $q=0$  blir begge lik 1. Hvis  $q > 0$ :

Sjeller når sanns. i a) < sanns. i b):

$$X - 4q^3(1-q) - q^4 < X - 3q^2(1-q) - q^3 \quad | : q^2$$

$$-4q(1-q) - q^2 < -3(1-q) - q$$

$$4q^2 - 4q - q^2 < 3q - 3 - q$$

$$3g^2 - 4g < 2g \Leftrightarrow$$

$$3g^2 - 6g + 3 < 0$$

$$3(g-1)^2 < 0 \quad \leftarrow \text{dette kan aldri inntreffe, } (g-1)^2 \geq 0$$

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Sanns. i a)  $\geq$  sanns. i b) for alle  $g$

(og likhet kun for  $g=0$  og  $g=1$ )

4.8.1  $X =$  antall sprakdadebitr i en till. velyt lyck.

Poisson-fordelt med forventet antall

$$\lambda = \frac{200}{40} = 5$$

$$\begin{aligned} \text{a)} \quad p(X \geq 4) &= 1 - p(0) - p(1) - p(2) - p(3) \\ &= 1 - e^{-5} \cdot \frac{5^0}{0!} - e^{-5} \cdot \frac{5^1}{1!} - e^{-5} \cdot \frac{5^2}{2!} - e^{-5} \cdot \frac{5^3}{3!} \\ &= 1 - e^{-5} \left( 1 + 5 + \frac{25}{2} + \frac{125}{6} \right) = 1 - \frac{118}{3} e^{-5} \approx 0,735 \end{aligned}$$

$$\text{b)} \quad p(X=0) = e^{-5} \cdot \frac{5^0}{0!} = e^{-5} = \underline{\underline{0.007}} \approx 0.7\%$$

4.8.2.

$X$  = antall bookede passasjerer som ikke møter

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Poisson - fordelt med  $\lambda = 4,04 \cdot \frac{1}{10^2} = \frac{4,04}{100} = \underline{4,04}$

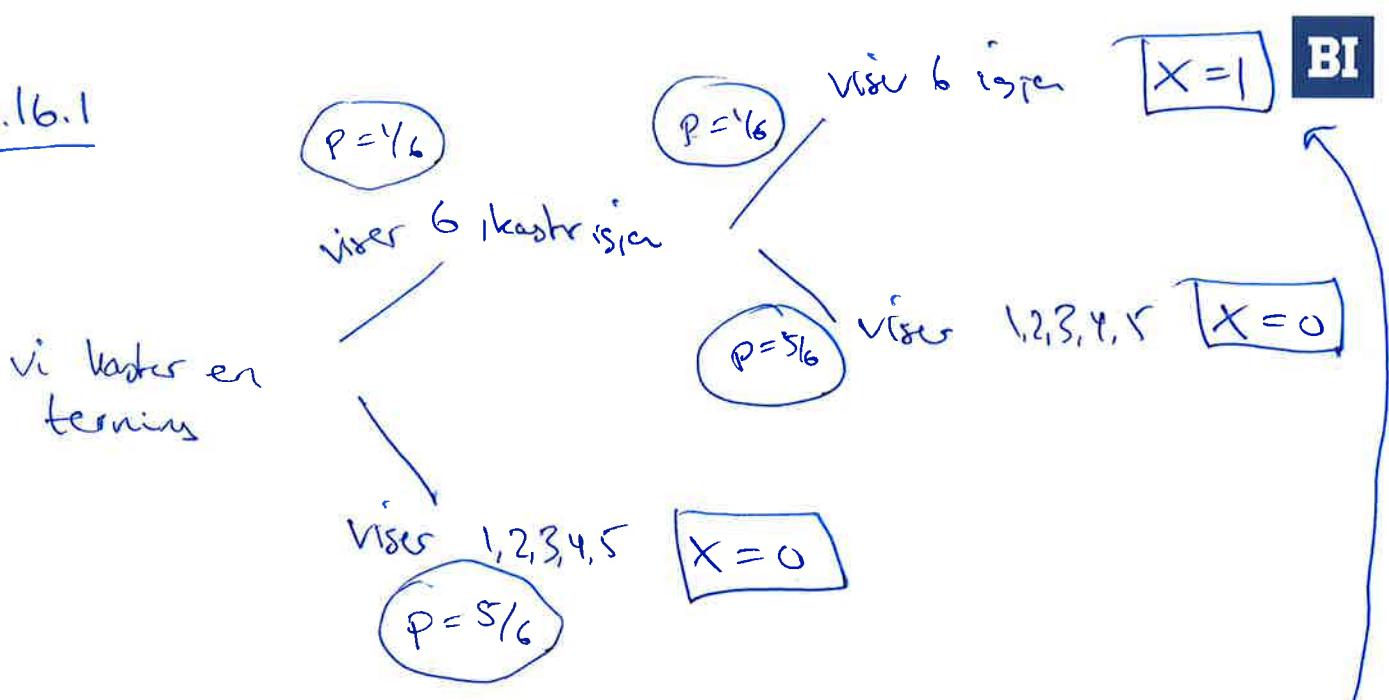
$$P(\text{overbooket}) = P(X < 4)$$

$$= p(0) + p(1) + p(2) + p(3)$$

$$= e^{-4,04} \cdot \left( 1 + 4,04 + \frac{4,04^2}{2} + \frac{4,04^3}{6} \right)$$

$$\simeq \underline{0,426} \simeq 42,6\%$$

4.16.1



For hver terning, er sannsynligheten for at 6 blir stående (etter to kast) lik  $\frac{1}{36} = \frac{1}{6} \cdot \frac{1}{6}$

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$X$  binomial  $(n, \frac{1}{36})$

$$E(X) = n \cdot p = \frac{n}{36}$$

$$\text{Var}(X) = n \cdot p \cdot (1-p) = \frac{n \cdot 1}{36} \cdot \frac{35}{36}$$

$$= \frac{35n}{36^2}$$

Blares formelen

$$E(X) = np \quad \text{Var}(X) = np(1-p)$$

for binomial  $X$

4.16.2

a)  $P(x) = c_i \cdot x \quad 1 \leq x \leq n$

$$P(1) = c_1 \quad P(2) = 2c_1 \quad P(3) = 3c_1 \quad \dots \quad P(n) = nc_1$$

Krav:

- i)  $P \geq 0$  for alle  $i$
- ii)  $P(1) + P(2) + \dots + P(n) = 1$

Krav i) betyr  $p(x) \geq 0 \Rightarrow c_1 \geq 0$ .

Da er  $p(2), p(3), \dots, p(n) \geq 0$  automatisch oppfyllt.

Krav ii):  $c_1 + 2c_1 + \dots + nc_1 = 1$

$$c_1 \cdot (1+2+3+\dots+n) = 1$$

$$c_1 \cdot \frac{n \cdot (n+1)}{2} = 1 \Rightarrow c_1 = \underline{\underline{\frac{2}{n(n+1)}}}$$

Konklusjon:

Sid  $c_1 = \frac{2}{n(n+1)} > 0$ , gir det en sannsynlighetsfordeling.

$c_1 \neq \frac{2}{n(n+1)}$  gir else — — —

b)  $p(x) = \frac{c_2}{x(x+1)}, x \geq 1$

$$p(1) = \frac{c_2}{2}, p(2) = \frac{c_2}{6}, p(3) = \frac{c_2}{12}, \dots$$

Krav i):  $c_2 \geq 0$

Krav ii):  $\frac{c_2}{2} + \frac{c_2}{6} + \frac{c_2}{12} + \dots = 1$

$$c_2 \cdot \left( \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \dots \right) = 1$$

↑  
må regne ut denne summen  
(venstreledd!)

Bruk at  $\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$  for to konstanter A, B:

$$\left. \begin{array}{l} \text{mult.} \\ \text{med} \\ x(x+1) \end{array} \right\} \rightarrow \begin{aligned} 1 &= A \cdot (x+1) + Bx &= Ax + A + Bx \\ 1 &= (A+B)x + A && \leftarrow \text{Sammenslutter} \\ &&&\text{konst.-ledd og } x\text{-ledd;} \end{aligned}$$

$$\boxed{\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}}$$

$$\left. \begin{aligned} 1 &= A \text{ og } 0 = A+B \\ A &= 1 & B &= -1 \end{aligned} \right\}$$

Spørre dette:

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$$\frac{1}{x} - \frac{1}{x+1} = \frac{1 \cdot (x+1)}{x \cdot (x+1)} - \frac{1 \cdot x}{(x+1) \cdot x} = \frac{(x+1) - x}{x(x+1)}$$

$$= \frac{1}{x(x+1)} ! \quad \text{oh.}$$

Der ned:  $\sum_{x=1}^{\infty} \frac{1}{x \cdot (x+1)} = \sum_{x=1}^{\infty} \left( \frac{1}{x} - \frac{1}{x+1} \right)$

$$= (1 - \frac{1}{2}) + (\frac{1}{2} - \frac{1}{3}) + (\frac{1}{3} - \frac{1}{4}) + \dots$$

$$\rightarrow y_2 \qquad \qquad y_6 \qquad \qquad y_{12}$$

$$= 1 + (-\cancel{\frac{1}{2}} + \cancel{\frac{1}{2}}) + (-\cancel{\frac{1}{3}} + \cancel{\frac{1}{3}}) + \dots$$

$$= \underline{\underline{1}} \qquad \text{Summen er } 1 !$$

Konklusjon: kraw ii):  $c_2 \cdot \left( \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \dots \right) = 1$

$$c_2 \cdot 1 = 1$$

$$\underline{\underline{c_2 = 1}}$$

oppfyller kraw i  
 $c_2 \geq 0$

Sannsynlighetsfordelingen er  $\underline{\underline{c_2 = 1}}$ .