

5.2.3. X = antall "heads" på n kast

binomial: $(n, \frac{1}{2})$

W = posisjon etter n kast (antall m. østover)

$$W = X - (n - X) = 2X - n.$$

antall	antall
flytt	flytt
østover	vestover

$$E(W) = 2E(X) - n = 2 \cdot np - n = n - n = 0$$

$$\text{Var}(W) = 2^2 \cdot E(X) = 4 \cdot n \cdot p(1-p) \quad \textcircled{p=\frac{1}{2}}$$

$$= 4 \cdot n \cdot \frac{1}{2} \cdot \frac{1}{2} = n$$

$$\Omega_W = \{-n, \dots, -2, -1, 0, 1, 2, \dots, n\}$$

$$p(W=i) = p(2X-n=i) = p(2X=i+n)$$

$$= p(X = \frac{i+n}{2})$$

$$(i = -n, \dots, -1, 0, 1, \dots, n) \quad = \frac{n!}{(\frac{i+n}{2})! (\frac{n-i}{2})!} \cdot \left(\frac{1}{2}\right)^{\frac{i+n}{2}} \cdot \left(\frac{1}{2}\right)^{1 - \frac{i+n}{2}}$$

när $i+n$ är partell

$$p(W=i) = p(X = \frac{i+n}{2}) = 0 \quad \text{när } i+n \text{ är oddetall}$$

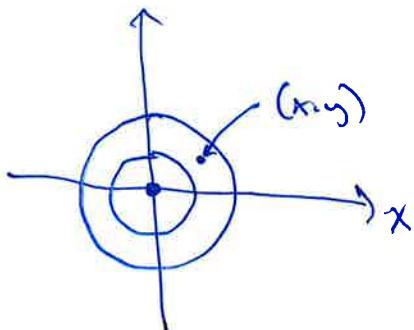
Elo: $n=3$

$p(W=3) = p(X=3) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$	$(3m \text{ øst})$
$p(W=2) = p(X=2,5) = 0$	$(2m \text{ øst})$
$p(W=1) = p(X=2) = 3 \cdot \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right) = \frac{3}{8}$	$(1m \text{ øst})$
\vdots	

5.2.4.

BI

Det kaster en pil på dartsblikk. Utfallset er posisjonen til treffpunktet, som kan måles ved koordinatene (x, y) som er to reelle tall.



$\Omega = \{(x, y) : x, y \text{ reelle tall}\}$
 Utfallsrommet er alle koordinater (x, y) - vendelig mange, og ikke tellbart.

$X = \text{Score}$ (dvs 10, 9, 8, ... ext. tall ved dobbelt eller trippel poeng etc)

Mulige verdier av $X =$
diskret mengde

5.3.3. X : sannsynlighetsfordeling \rightarrow

$$Y : Y = a + bX$$

$$\begin{cases} p(x) = p_X(x) \\ p(y) = p_Y(y) \end{cases}$$

$$\begin{aligned} p(Y=y) &= p(a+bX=y) = p(bX=y-a) \\ &\stackrel{\uparrow}{=} p\left(X=\frac{y-a}{b}\right) \\ &\underline{p_Y(y) = p_X\left(\frac{y-a}{b}\right)} \end{aligned}$$

$$\begin{aligned} F_Y(y) &= p(Y \leq y) = p(a+bX \leq y) = p(bX \leq y-a) \\ &= p\left(X \leq \frac{y-a}{b}\right) = F_X\left(\frac{y-a}{b}\right) \quad \text{når } b > 0 \end{aligned}$$

Når $b < 0$: Snar likeheten ved divisjon på b :

$$F_Y(y) = P(bX \leq y-a) = P(X \geq \frac{y-a}{b})$$

$$= 1 - P(X < \frac{y-a}{b})$$

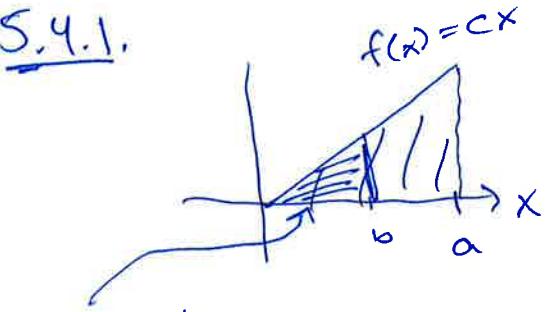
Hvis $\frac{y-a}{b} < 0$, $b < 0$

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Dette blir $\frac{1 - F_X(\frac{y-a}{b})}{\text{Hvis } \frac{y-a}{b} \text{ ikke er heltall,}}$
og ellers ~~$F_X(\frac{y-a}{b})$ ikke er det største tall~~

$$\frac{1 - F_X(\frac{y-a}{b} - 1)}{\text{(hvis } \frac{y-a}{b} \text{ er et heltall)}}.$$

5.4.1.



$$F(b) = \int_0^b f(x) dx$$

$$= \int_0^b \frac{2}{a^2} \cdot x dx$$

$$= \frac{2}{a^2} \cdot \left[\frac{1}{2} x^2 \right]_0^b = \frac{1}{a^2} \cdot (b^2 - 0^2) = \underline{\underline{\frac{b^2}{a^2}}}, \quad 0 < b < a$$

i) $f(x) \geq 0 \quad c \geq 0$

ii) $\int_0^a f(x) dx = 1$
" "

$$\int_0^a c x dx = \left[c \cdot \frac{1}{2} x^2 \right]_0^a$$

$$= c \cdot \frac{1}{2} a^2 - c \cdot 0$$

$$= \frac{c}{2} a^2 = 1$$

$$\Rightarrow c = \underline{\underline{\frac{2}{a^2}}} > 0$$

5.4.2. $f_1(x), f_2(x)$ teiltatsächlich
ein

$$0 \leq x \leq 1$$

i) $\lambda \cdot f_1(x) + (1-\lambda) f_2(x) \geq 0 ? \quad \leftarrow \text{oh.}$

$$x \geq 0, f_1(x) \geq 0 \rightarrow \lambda f_1(x) \geq 0$$

$$x \leq 1, f_2(x) \geq 0 \rightarrow (1-\lambda) \geq 0, (1-\lambda) f_2(x) \geq 0$$

ii) $\int_{-\infty}^{\infty} \lambda \cdot f_1(x) + (1-\lambda) f_2(x) dx = \lambda \cdot \int_{-\infty}^{\infty} f_1(x) dx$
" "
 $+ (1-\lambda) \cdot \int_{-\infty}^{\infty} f_2(x) dx = \lambda \cdot 1 + (1-\lambda) \cdot 1 = \lambda + 1-\lambda$
 $\underbrace{= 1}_{\text{oh.}}$

Konkl: $\lambda \cdot f_1(x) + (1-\lambda) f_2(x)$ teiltatsächl. nur $0 \leq x \leq 1$.

5.4.3 $F_1(x), F_2(x)$ kumulative sanns. fordelinger

$$\Rightarrow f_1(x) = F'_1(x), f_2(x) = F'_2(x) \quad \text{tettfetstikkj. i m}$$

$$\Rightarrow \lambda \cdot f_1(x) + (1-\lambda) f_2(x) \quad \text{tettfetstikkj. for } 0 \leq \lambda \leq 1 \\ (\text{pga. 5.4.2})$$

$$\Rightarrow \lambda F_1(b) + (1-\lambda) F_2(b) = \int_{-\infty}^b \lambda f_1(x) + (1-\lambda) f_2(x) dx \\ \text{er alk. sannsynlighetsfordeling, med tettfet}$$

$$[\lambda F_1(x) + (1-\lambda) F_2(x)]' = \lambda f_1(x) + (1-\lambda) f_2(x)$$

5.5.1. X std. normal fordelt:

$$Y = X^2$$

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

$$F_Y(y) = P(Y \leq y) = P(X^2 \leq y) = P(-\sqrt{y} \leq X \leq \sqrt{y}) \\ = F_X(\sqrt{y}) - F_X(-\sqrt{y})$$

||

$$f(y) = (F_Y(y))'_y = \frac{1}{2\sqrt{y}} \cdot f_X(\sqrt{y}) - \frac{1}{-2\sqrt{y}} \cdot f_X(-\sqrt{y})$$

$$= \frac{1}{2\sqrt{y}} (f_X(\sqrt{y}) + f_X(-\sqrt{y}))$$

$$= \frac{1}{2\sqrt{y}} \left(\frac{1}{\sqrt{2\pi}} e^{-y^2/2} + \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \right) = \underline{\underline{\frac{1}{\sqrt{y}} \cdot \frac{1}{\sqrt{2\pi}} e^{-y^2/2}}} \\ (\text{for } y > 0)$$

S.5.4.

$$f(x) = 6x(1-x), \quad 0 \leq x \leq 1 \quad \leftarrow X$$

$$f_Y(y) = ? \quad \leftarrow Y = 1 - x$$

$$F_Y(y) = P(Y \leq y) = P(1-x \leq y) = P(1-y \leq x)$$

$$= P(X \geq 1-y) = 1 - P(X \leq 1-y) = 1 - F_X(1-y)$$

$$f_Y(y) = (F_Y(y))' = (1 - F_X(1-y))' = -F_X'(1-y) \cdot (-1)$$

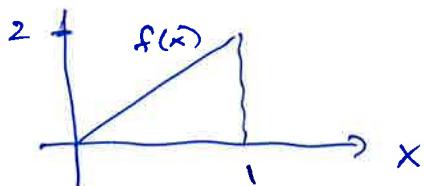
$$= f_X(1-y) = 6(1-y) \cdot (1-(1-y))$$

$$= 6(1-y)y = \underline{6y(1-y)}, \quad 0 \leq y \leq 1$$

(same distribution
as X)

S.6.1.

$$f(x) = 2x, \quad 0 \leq x \leq 1$$



$$\begin{aligned} E(X) &= \int_{-\infty}^{\infty} x \cdot f(x) dx \\ &= \int_0^1 x \cdot 2x dx \end{aligned}$$

$$= \int_0^1 2x^2 dx = \left[\frac{2}{3}x^3 \right]_0^1$$

$$= \frac{2}{3} - 0 = \underline{\underline{\frac{2}{3}}}$$

5.12.5. Sian $\text{Var}(x) = E(x^2) - E(x)^2 \geq 0$,
 s.a. es $E(x^2) \geq E(x)^2$.

5.12.6.

a) i) $f(x) \geq 0$ oh havis $c \geq 0$

$$\text{ii)} \int f(x) dx = 1$$

$$\int_0^1 cx(1-x) dx = \int_0^1 cx - cx^2 dx$$

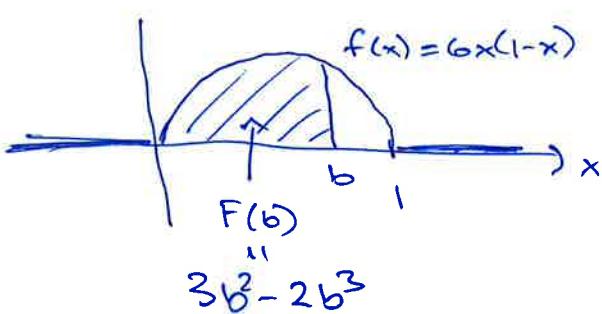
$$= c \cdot \left[\frac{1}{2}x^2 - \frac{1}{3}x^3 \right]_0^1 = c \cdot \left(\frac{1}{2} - \frac{1}{3} \right) = c \cdot \frac{1}{6} = 1$$

$$c = 6 > 0 \text{ oh.}$$

Tætthet for $c=6$.

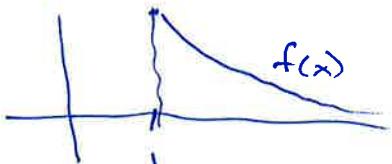
$$F(b) = \int_0^b 6x(1-x) dx = 6 \left[\frac{1}{2}x^2 - \frac{1}{3}x^3 \right]_0^b = 6 \left(\frac{b^2}{2} - \frac{b^3}{3} \right)$$

$$= 3b^2 - 2b^3, \quad b \in [0,1]$$



$$\begin{cases} F(b) = 0, & b < 0 \\ F(b) = 1, & b > 1 \end{cases}$$

b)



$$\begin{aligned} \int_1^\infty \frac{1}{x} dx &= c \cdot [\ln x]_1^\infty \\ &= c \cdot (\lim_{b \rightarrow \infty} \ln b - \ln 1) \\ &= \infty \quad \text{for } c > 0 \end{aligned}$$

i) ikke fordelig

c) $f(x) = C \cdot e^{-x^2 + 4x}$, $x \in (-\infty, \infty)$

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$$\left. \begin{aligned} -x^2 + 4x &= -(x^2 - 4x) = -(x^2 - 4x + 4) + 4 \\ &= -(x-2)^2 + 4 \end{aligned} \right\} \begin{array}{l} f(x) \text{ ligger} \\ \text{på} \\ \text{normalford.} \\ \text{forsøker} \\ \text{at få den} \\ \text{på riktig} \\ \text{form} \end{array}$$

||

$$\begin{aligned} f(x) &= C \cdot e^{-x^2 + 4x} = C \cdot e^{-(x-2)^2 + 4} \\ &= C \cdot e^4 \cdot e^{-\frac{(x-2)^2}{1}} = \frac{1}{\sqrt{2\pi} \cdot \sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \end{aligned}$$

$$F_x(x) = F_X(x)$$

\leftarrow alk. fordeling
for normalford.

$$= G\left(\frac{x-2}{\sqrt{2}}\right) = \underline{\underline{G(\sqrt{2}(x-2))}}$$

$G =$ alk. fordeling
for std.
normalford.

(-skrivet \varnothing i)
boken

ser at dette er
normalfordeling med
 $\mu = 2$, $\sigma = \frac{1}{\sqrt{2}}$

hvis

$$C \cdot e^4 = \frac{1}{\sqrt{2\pi} \cdot \sigma}$$

$$C \cdot e^4 = \frac{1}{\sqrt{2\pi} \cdot \sqrt{\frac{1}{2}}} = \frac{1}{\sqrt{\pi}}$$

$$C = \underline{\underline{e^{-4} \cdot \frac{1}{\sqrt{\pi}}}}$$

d) $f(x) = C \frac{e^x}{(1+e^x)^2}$ - $x \in (-\infty, \infty)$

i) $C \geq 0$ gir $f(x) \geq 0$

ii) $\int_{-\infty}^{\infty} f(x) dx = 1$

$$C \cdot \int_{-\infty}^{\infty} \frac{e^x}{(1+e^x)^2} dx = 1$$

$$\int_{-\infty}^{\infty} \frac{e^x}{(1+e^x)^2} dx = \int_1^{\infty} \frac{1}{u^2} du$$

$$= \left[-\frac{1}{u} \right]_1^{\infty} = 0 + \frac{1}{1} = 1$$

$$\underline{\underline{C = 1}}$$

$$\begin{aligned}
 F_X(b) &= \int_{-\infty}^b \frac{e^x}{(1+e^x)^2} dx = \int_1^{1+e^b} \frac{1}{u^2} du \\
 &= \left[-\frac{1}{u} \right]_1^{1+e^b} = -\frac{1}{1+e^b} + 1 = 1 - \frac{1}{1+e^b} \\
 &= \underline{\underline{\frac{e^b}{e^b + 1}}}
 \end{aligned}$$

S.12.26. Anta $\text{Var}(x) = 0$, og da $a = E(x)$.

Da har vi:

$$\text{Var}(x) = \int_{-\infty}^{\infty} (x-a)^2 \cdot f(x) dx = 0$$

Siden $f(x) \geq 0$ for alle x , følger det at
 $(x-a)^2 \geq 0 \quad \rightarrow, -$

fordi anden $(x-a)^2 \cdot f(x) = 0$ for alle x

Det betyr at når $x \neq a$, så er $(x-a)^2 \neq 0$

$$\Rightarrow f(x) = 0$$

$\Rightarrow f(x) = 0$ for $x \neq a$. Da er x diskret,
med $p(\underline{x=a}) = 1$.