Department of Economics

Written examination in:	**GRA 60353 Mathematics**
Examination date:	10.12.2010, 09:00 – 12:00
Permitted examination aids:	Bilingual dictionary BI-approved exam calculator: TEXAS INSTRUMENTS BA II Plus™
Answer sheets:	Squares
Total number of pages:	2

Question 1.

We consider the function $f(x, y, z) = x^2e^x + yz - z^3$.

(a) Find all stationary points of f.
(b) Compute the Hessian matrix of f. Classify the stationary points of f as local maxima, local minima or saddle points.

Question 2.

We consider the matrix A and the vector $\mathbf{v}$ given by

$$A = \begin{pmatrix} 1 & 7 & -2 \\ 0 & s & 0 \\ 1 & 1 & 4 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

(a) Compute the determinant and the rank of A.
(b) Find all eigenvalues of A. Is $\mathbf{v}$ an eigenvector for A?
(c) Determine the values of s such that A is diagonalizable.

Question 3.

(a) You borrow an amount K. The interest rate per period is r. The repayment is 500 in the first period, and increases with 10 for each subsequent period. Show that the outstanding balance b_t after period t satisfies the difference equation

$$b_{t+1} = (1 + r)b_t - (500 + 10t), \quad b_0 = K$$

and solve this difference equation.
(b) Find the general solution of the differential equation $y'' + y' - 6y = te^t$.
(c) Solve the initial value problem

$$\frac{t}{y^2}\, y' = \frac{1}{y} - 3t^2, \quad y(1) = \frac{1}{3}$$

Question 4.

We consider the function $f(x, y, z) = xyz$.

(a) The function g is defined on the set $D = \{(x, y, z) : x > 0, y > 0, z > 0\}$, and it is given by

$$g(x, y, z) = \frac{1}{f(x, y, z)} = \frac{1}{xyz}$$

Is g a convex or concave function on D?

(b) Maximize $f(x, y, z)$ subject to $x^2 + y^2 + z^2 \leq 1$.