BI NORWEGIAN SCHOOL OF MANAGEMENT

Department of Economics

Multiple-choice examination in:	**GRA 60352 Mathematics**
Examination date:	24.09.2010, 14:00 – 15:00
Permitted examination aids:	Bilingual dictionary.
	BI-approved exam calculator: TEXAS INSTRUMENTS BA II Plus^{TM}
Answer sheets:	Answer sheet for multiple choice examinations
Total number of pages:	4
Number of attachments:	1 (example of how to use the answer sheet)

PLEASE READ THE FOLLOWING BEFORE YOU BEGIN!

- Students must themselves assure that the examination papers are complete.
- Students must provide the following information on the answer sheet:
  - Examination code
  - Personal initials
  - ID number

  The student registration number must be recorded with both the appropriate numbers and by putting an "X" by the corresponding number in the columns below.
- Pens with green ink and pencils cannot be used in filling in answer sheets. Answer sheets must not be used for writing rough drafts.
- **All answers must be recorded with an "X" under the letter you believe corresponds with the correct answer.**
- **Cancel an "X" by filling in the box completely (boxes that are completely filled in will not be registered). "X" in two boxes for one question will be registered as a wrong answer.**
- The attached example shows you how the answer sheet would be filled in if A were the correct answer for question 1, B correct for question 2, C correct for question 3 and D correct for question 4. An "X" under E indicates that you choose not to answer question 5.
- **Your answers are to be recorded on the answer sheet. Answers written on the examination papers and not on the answer sheets will not be graded.**
- There is only <u>one</u> right answer for each question. Because the questions are weighted equally, it can be to your advantage to answer the simplest questions first.
- Wrong answers are given -1 point, unanswered questions get 0 points (indicated by an "X" next to E") and correct answers are given 3 points.
- You can keep the examination papers.

Good luck!

This exam has 8 questions

Question 1.

Consider the linear system

$$\begin{pmatrix} 1 & -3 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -4 \\ 0 & 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 4 \\ 0 \end{pmatrix}$$

Which statement is true?

(A) The linear system is inconsistent.
(B) The linear system has a unique solution.
(C) The linear system has one degree of freedom
(D) The linear system has two degrees of freedom
(E) I prefer not to answer.

Question 2.

Consider the set of vectors $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, where

$$\mathbf{v}_1 = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 0 \\ 1 \\ h \end{pmatrix}$$

and h is a parameter. **Which statement is true?**

(A) $\mathcal{B}$ is a linearly independent set of vectors for all h
(B) $\mathcal{B}$ is a linearly independent set of vectors exactly when $h = 3$
(C) $\mathcal{B}$ is a linearly independent set of vectors exactly when $h \neq 1/7$
(D) $\mathcal{B}$ is a linearly independent set of vectors exactly when $h \neq 3$
(E) I prefer not to answer.

Question 3.

Compute the rank of the matrix

$$A = \begin{pmatrix} 2 & 5 & -3 & -4 & 8 \\ 4 & 7 & -4 & -3 & 9 \\ 6 & 9 & -5 & -2 & 4 \end{pmatrix}$$

Which statement is true?

(A) $\operatorname{rk} A = 1$
(B) $\operatorname{rk} A = 2$
(C) $\operatorname{rk} A = 3$
(D) $\operatorname{rk} A = 4$
(E) I prefer not to answer.

Question 4.

Consider the matrix

$$A = \begin{pmatrix} 1 & 2 \\ 4 & -1 \end{pmatrix}$$

Which statement is true?

(A) A has eigenvalues $\lambda = 1$ and $\lambda = -1$
(B) A has eigenvalues $\lambda = 2$ and $\lambda = 4$
(C) A has a single eigenvalue $\lambda = 3$
(D) A has eigenvalues $\lambda = 3$ and $\lambda = -3$
(E) I prefer not to answer.

Question 5.

Consider the matrix

$$A = \begin{pmatrix} 5 & 4 \\ -1 & 9 \end{pmatrix}$$

Which statement is true?

(A) $\lambda = 7$ is not an eigenvalue for A
(B) $\lambda = 7$ is an eigenvalue for A and A is diagonalizable
(C) $\lambda = 7$ is an eigenvalue for A but A is not diagonalizable
(D) $\lambda = 7$ and $\lambda = 3$ are eigenvalues for A
(E) I prefer not to answer.

Question 6.

Consider the quadratic form

$$Q(x_1, x_2) = x_1^2 - 4x_1x_2 + 4x_2^2$$

Which statement is true?

(A) Q is positive semidefinite but not positive definite
(B) Q is negative semidefinite but not negative definite
(C) Q is indefinite
(D) Q is positive definite
(E) I prefer not to answer.

Question 7.

Consider the function

$$f(x_1, x_2, x_3) = 3x_1^2 + 2x_1x_2 + 3x_2^2 + x_3^2 + x_1 - x_2$$

defined on $\mathbb{R}^3$. **Which statement is true?**

(A) f is a convex function but not a concave function
(B) f is a convex function and a concave function
(C) f is not a convex function but a concave function
(D) f is neither a convex nor a concave function
(E) I prefer not to answer.

Question 8.

A car rental agency has two rental locations in a certain city, one downtown and one at the airport. Let x_t denote the number of cars at the downtown location after t days, and y_t the number of cars at the airport after t days. We assume that the distribution of cars is given by the following model:

$$\begin{pmatrix} x_{t+1} \\ y_{t+1} \end{pmatrix} = A \cdot \begin{pmatrix} x_t \\ y_t \end{pmatrix}, \quad \text{where } A = \begin{pmatrix} 0.97 & 0.02 \\ 0.03 & 0.98 \end{pmatrix}$$

A steady state for this system is a state $\mathbf{v} = \left(\begin{smallmatrix} x \\ y \end{smallmatrix}\right)$ such that $A\mathbf{v} = \mathbf{v}$. **Which statement is true?**

(A) The system has a steady state with 30% of the cars at the downtown location
(B) The system has a steady state with 40% of the cars at the downtown location
(C) The system has a steady state with 50% of the cars at the downtown location
(D) The system has a steady state with 60% of the cars at the downtown location
(E) I prefer not to answer.