Problem Sheet 3 with Solutions
GRA 6035 Mathematics

BI Norwegian Business School

Problems

1. Express the vector $\mathbf{w}$ as a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$ when

$$\mathbf{w} = \begin{pmatrix} 8 \\ 9 \end{pmatrix}, \quad \mathbf{v}_1 = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

Draw the three vectors in a two-dimensional coordinate system.

2. Determine if the following pairs of vectors are linearly independent:

$$\text{(a)} \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ -6 \end{pmatrix} \quad \text{(b)} \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad \text{(c)} \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Draw the vectors in the plane in each case and explain geometrically.

3. Show that the following vectors are linearly dependent:

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$$

4. Assume that $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are linearly independent m-vectors.

a) Show that $\mathbf{a}+\mathbf{b}, \mathbf{b}+\mathbf{c}$ and $\mathbf{a}+\mathbf{c}$ are linearly independent.
b) Is the same true of $\mathbf{a}-\mathbf{b}, \mathbf{b}+\mathbf{c}$ and $\mathbf{a}+\mathbf{c}$?

5. Let $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ be m-vectors. Show that at least one of the vectors can be written as a linear combinations of the others if and only if the vector equation

$$x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + \cdots + x_n\mathbf{v}_n = \mathbf{0}$$

has non-trivial solutions.

6. Prove that the following vectors are linearly independent:

$$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

7. Using the definition of rank of a matrix, prove that any set of n vectors in $\mathbb{R}^m$ must be linearly dependent if $n > m$.

8. Show that the vectors

$$\begin{pmatrix} 3 \\ 4 \\ -1 \\ 2 \end{pmatrix} \text{ and } \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

are linearly independent by computing a minor of order two.

9. We consider the column vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ of the matrix

$$\begin{pmatrix} 1 & 2 & 4 \\ 3 & 7 & 0 \\ 5 & 11 & 8 \end{pmatrix}$$

a) Show that the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly dependent.
b) Find a non-trivial solution of $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = \mathbf{0}$, and use this solution to express one of the three vectors as a linear combination of the two others.

10. We consider the column vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ of the matrix

$$\begin{pmatrix} 1 & 2 & 4 & -1 \\ 3 & 7 & 0 & 4 \\ 5 & 11 & 8 & 2 \\ -2 & -5 & 4 & -5 \end{pmatrix}$$

a) Show that the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ are linearly dependent.
b) Are there three linearly independent vectors among these four vectors? If not, what about two linearly independent vectors among them?

11. Let A be any matrix. The *Gram matrix* of A is the square matrix A^TA. A theorem states that for any matrix A, we have that

$$\text{rk}(A) = \text{rk}(A^TA)$$

Use this theorem to find the rank of the following matrices:

$$a) \quad A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{pmatrix} \qquad\qquad b) \quad A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 6 \end{pmatrix}$$

12. Let A be any $m \times n$-matrix. We define the *null space* of A to be the collection of all n-vectors $\mathbf{x}$ such that $A\mathbf{x} = \mathbf{0}$. In other words, the null space of A is the solution set of the homogeneous linear system $A\mathbf{x} = \mathbf{0}$.

a) Show that the null space of A is equal to the null space of the Gram matrix A^TA.
b) Prove that $\text{rk}(A) = \text{rk}(A^TA)$.

13. Midterm Exam in GRA6035 24/09/2010, Problem 2
Consider the set of vectors $\mathscr{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, where

$$\mathbf{v}_1 = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 0 \\ 1 \\ h \end{pmatrix}$$

and h is a parameter. **Which statement is true?**

a) $\mathscr{B}$ is a linearly independent set of vectors for all h
b) $\mathscr{B}$ is a linearly independent set of vectors exactly when $h = 3$
c) $\mathscr{B}$ is a linearly independent set of vectors exactly when $h \neq 1/7$
d) $\mathscr{B}$ is a linearly independent set of vectors exactly when $h \neq 3$
e) I prefer not to answer.

14. Mock Midterm Exam in GRA6035 09/2010, Problem 1
Consider the vector $\mathbf{w}$ and the set of vectors $\mathscr{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, where

$$\mathbf{w} = \begin{pmatrix} -4 \\ 3 \\ h \end{pmatrix}, \quad \mathbf{v}_1 = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 5 \\ -4 \\ -7 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$$

Which statement is true?

a) $\mathbf{w}$ is a linear combination of the vectors in $\mathscr{B}$ for all values of h
b) $\mathbf{w}$ is a linear combination of the vectors in $\mathscr{B}$ exactly when $h \neq 5$
c) $\mathbf{w}$ is a linear combination of the vectors in $\mathscr{B}$ exactly when $h = 5$
d) $\mathbf{w}$ is not a linear combination of the vectors in $\mathscr{B}$ for any value of h
e) I prefer not to answer.

15. Midterm Exam in GRA6035 24/05/2011, Problem 3
Consider the set of vectors $\mathscr{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, where

$$\mathbf{v}_1 = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} h+1 \\ h \\ h-2 \end{pmatrix}$$

and h is a parameter. **Which statement is true?**

a) $\mathscr{B}$ is a linearly independent set of vectors for all h
b) $\mathscr{B}$ is a linearly independent set of vectors exactly when $h = 0$
c) $\mathscr{B}$ is a linearly independent set of vectors exactly when $h \neq 5$
d) $\mathscr{B}$ is a linearly independent set of vectors exactly when $h \neq -1$
e) I prefer not to answer.

Solutions

1 We must find numbers c_1 and c_2 so that

$$\begin{pmatrix}8\\9\end{pmatrix} = c_1\begin{pmatrix}2\\5\end{pmatrix} + c_2\begin{pmatrix}-1\\3\end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix}2 & -1\\5 & 3\end{pmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix} = \begin{pmatrix}8\\9\end{pmatrix}$$

Multiplying with the inverse from the left, we get that

$$\begin{pmatrix}c_1\\c_2\end{pmatrix} = \begin{pmatrix}2 & -1\\5 & 3\end{pmatrix}^{-1}\begin{pmatrix}8\\9\end{pmatrix} = \frac{1}{11}\begin{pmatrix}3 & 1\\-5 & 2\end{pmatrix}\begin{pmatrix}8\\9\end{pmatrix} = \begin{pmatrix}3\\-2\end{pmatrix}$$

Hence $c_1 = 3$ and $c_2 = -2$.

2 We form the 2×2-matrix with the pair of vectors as columns in each case. We know that the vectors are linearly independent if and only if the determinant of the matrix is non-zero; therefore we compute the determinant in each case:

$$a) \quad \begin{vmatrix}-1 & 3\\2 & -6\end{vmatrix} = 0 \qquad b) \quad \begin{vmatrix}2 & 3\\-1 & 4\end{vmatrix} = 11 \neq 0 \qquad c) \quad \begin{vmatrix}-1 & 1\\1 & -1\end{vmatrix} = 0$$

It follows that the vectors in b) are linearly independent, while the vectors in a) and the vectors in c) are linearly dependent.

An alternative solution is to consider the vector equation $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 = \mathbf{0}$ for each pair of vectors $\mathbf{v}_1, \mathbf{v}_2$ and solve the equation. The equation has non-trivial solutions (one degree of freedom) in a) and c), but only the trivial solution $c_1 = c_2 = 0$ in b). The conclusion is therefore the same as above.

3 We consider the vector equation

$$c_1\begin{pmatrix}1\\1\\1\end{pmatrix} + c_2\begin{pmatrix}2\\1\\0\end{pmatrix} + c_3\begin{pmatrix}3\\1\\4\end{pmatrix} + c_4\begin{pmatrix}1\\2\\-2\end{pmatrix} = \begin{pmatrix}1 & 2 & 3 & 1\\1 & 1 & 1 & 2\\1 & 0 & 4 & -2\end{pmatrix}\begin{pmatrix}c_1\\c_2\\c_3\\c_4\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

We use elementary row operations to simplify the coefficient matrix of this linear system:

$$\begin{pmatrix}1 & 2 & 3 & 1\\1 & 1 & 1 & 2\\1 & 0 & 4 & -2\end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix}1 & 2 & 3 & 1\\0 & -1 & -2 & 1\\0 & -2 & 1 & -3\end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix}1 & 2 & 3 & 1\\0 & -1 & -2 & 1\\0 & 0 & 5 & -5\end{pmatrix}$$

We see that the linear system has one degree of freedom, and therefore there are non-trivial solutions. Hence the vectors are linearly dependent.

An alternative solution is to argue that the coefficient matrix can maximally have rank 3, and therefore must have at least one degree of freedom.

4 For the vectors in a), we consider the vector equation

$$c_1(\mathbf{a}+\mathbf{b})+c_2(\mathbf{b}+\mathbf{c})+c_3(\mathbf{a}+\mathbf{c})=\mathbf{0} \quad \Leftrightarrow \quad (c_1+c_3)\mathbf{a}+(c_1+c_2)\mathbf{b}+(c_2+c_3)\mathbf{c}=\mathbf{0}$$

Since $\mathbf{a},\mathbf{b},\mathbf{c}$ are linearly independent, this gives the equations

$$\begin{aligned} c_1 \quad\; + c_3 &= 0 \\ c_1 + c_2 \quad\; &= 0 \\ c_2 + c_3 &= 0 \end{aligned} \quad \Leftrightarrow \quad \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

But the determinant of the coefficient matrix is

$$\begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 1(1)-1(-1)=2\neq 0$$

so the only solution is the trivial solutions. Hence the vectors are linearly independent. For the vectors in b), we consider the vector equation

$$c_1(\mathbf{a}-\mathbf{b})+c_2(\mathbf{b}+\mathbf{c})+c_3(\mathbf{a}+\mathbf{c})=\mathbf{0} \quad \Leftrightarrow \quad (c_1+c_3)\mathbf{a}+(-c_1+c_2)\mathbf{b}+(c_2+c_3)\mathbf{c}=\mathbf{0}$$

Since $\mathbf{a},\mathbf{b},\mathbf{c}$ are linearly independent, this gives the equations

$$\begin{aligned} c_1 \quad\; + c_3 &= 0 \\ -c_1 + c_2 \quad\; &= 0 \\ c_2 + c_3 &= 0 \end{aligned} \quad \Leftrightarrow \quad \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

But the determinant of the coefficient matrix is

$$\begin{vmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 1(1)-(-1)(-1)=0$$

so there is at least one degree of freedom and non-trivial solutions. Hence the vectors are linearly dependent.

5 Let $\mathbf{v}_1,\mathbf{v}_2,\dots,\mathbf{v}_n$ be m-vectors. We consider the following statements:

a) At least one of the vectors can be written as a linear combinations of the others.
b) The vector equation $x_1\mathbf{v}_1+x_2\mathbf{v}_2+\cdots+x_n\mathbf{v}_n=\mathbf{0}$ has non-trivial solutions.

We must show that $(a)\Leftrightarrow(b)$. If statement (a) holds, then one of the vectors, say $\mathbf{v}_n$, can be written as a linear combination of the other vectors,

$$\mathbf{v}_n=c_1\mathbf{v}_1+c_2\mathbf{v}_2+\cdots+c_{n-1}\mathbf{v}_{n-1}$$

This implies that

$$c_1\mathbf{v}_1+c_2\mathbf{v}_2+\cdots+c_{n-1}\mathbf{v}_{n-1}+(-1)\mathbf{v}_n=\mathbf{0}$$

which is a non-trivial solution (since $c_n = -1 \neq 0$). So (a) implies (b). Conversely, if (b) holds, then there is a non-trivial solution to the vector equation

$$c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_{n-1}\mathbf{v}_{n-1} + c_n\mathbf{v}_n = \mathbf{0}$$

So at least one of the variables are non-zero; say $c_n \neq 0$. Then we can rewrite this vector equation as

$$\mathbf{v}_n = -\frac{1}{c_n}(c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_{n-1}\mathbf{v}_{n-1}$$

Hence one of the vectors can be written as a linear combination of the others, and (b) implies (a).

6 We compute the determinant

$$\begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 1(1) + 1(2) = 3$$

Since the determinant is non-zero, the vectors are linearly independent.

7 We consider n vectors in $\mathbb{R}^m$, and form the $m \times n$-matrix with these vectors as columns. Since $n > m$, the maximal rank that the matrix can have is m, and therefore the corresponding homogeneous linear system has at least $n - m > 0$ degrees of freedom. Therefore, the vectors must be linearly dependent.

8 We form the matrix with the two vectors as columns, and compute the 2-minor obtained by deleting the last two rows. We get

$$\begin{vmatrix} 3 & 0 \\ 4 & 1 \end{vmatrix} = 3 \neq 0$$

Hence the matrix has rank two, and there are no degrees of freedom. Therefore, the vectors are linearly independent.

9 We reduce the matrix to an echelon form using elementary row operations:

$$\begin{pmatrix} 1 & 2 & 4 \\ 3 & 7 & 0 \\ 5 & 11 & 8 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & -12 \\ 0 & 1 & -12 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & -12 \\ 0 & 0 & 0 \end{pmatrix}$$

We see that the vector equation $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = \mathbf{0}$ has one degree of freedom, and it follows that the vectors are linearly dependent. Moreover, we see that x_3 is a free variable and that the solutions are given by

$$x_2 = 12x_3, \quad x_1 = -2(12x_3) - 4x_3 = -28x_3$$

In particular, one solution is $x_1 = -28$, $x_2 = 12$, $x_3 = 1$. This implies that

$$-28\mathbf{v}_1 + 12\mathbf{v}_2 + \mathbf{v}_3 = \mathbf{0} \quad \Rightarrow \quad \mathbf{v}_3 = 28\mathbf{v}_1 - 12\mathbf{v}_2$$

10 We use elementary row operations to reduce the matrix to an echelon form:

$$\begin{pmatrix} 1 & 2 & 4 & -1 \\ 3 & 7 & 0 & 4 \\ 5 & 11 & 8 & 2 \\ -2 & -5 & 4 & -5 \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} 1 & 2 & 4 & -1 \\ 0 & 1 & -12 & 7 \\ 0 & 1 & -12 & 7 \\ 0 & -1 & 12 & -7 \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} 1 & 2 & 4 & -1 \\ 0 & 1 & -12 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

We see that there are are two degrees of freedom, since the matrix has rank 2. Therefore, the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ are linearly dependent, and there are not three linearly independent vectors among them either. However, there are two linearly independent column vectors since the rank is two. In fact, $\mathbf{v}_1$ and $\mathbf{v}_2$ are linearly independent since there are pivot positions in column 1 and 2.

11 We compute the Gram matrix A^TA and its determinant in each case: For the matrix in a) we have

$$A^TA = \begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{pmatrix}^T \begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{pmatrix} = \begin{pmatrix} 30 & 60 \\ 60 & 120 \end{pmatrix} \quad \Rightarrow \quad \det(A^TA) = \begin{vmatrix} 30 & 60 \\ 60 & 120 \end{vmatrix} = 0$$

This means that $\mathrm{rk}(A) < 2$, and we see that $\mathrm{rk}(A) = 1$ since $A \neq 0$. For the matrix in b) we have

$$A^TA = \begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 6 \end{pmatrix}^T \begin{pmatrix} 1 & 2 \\ 2 & -1 \\ 3 & 6 \end{pmatrix} = \begin{pmatrix} 14 & 18 \\ 18 & 41 \end{pmatrix} \quad \Rightarrow \quad \det(A^TA) = \begin{vmatrix} 14 & 18 \\ 18 & 41 \end{vmatrix} = 250$$

This means that $\mathrm{rk}(A) = 2$.

12 To show that the null space of A equals the null space of A^TA, we have to show that $A\mathbf{x} = \mathbf{0}$ and $A^TA\mathbf{x} = \mathbf{0}$ have the same solutions. It is clear that multiplication with A^T from the left gives

$$A\mathbf{x} = \mathbf{0} \quad \Rightarrow \quad A^TA\mathbf{x} = \mathbf{0}$$

so any solution of $A\mathbf{x} = \mathbf{0}$ is also a solution of $A^TA\mathbf{x} = \mathbf{0}$. Conversely, suppose that $\mathbf{x}$ is a solution of $A^TA\mathbf{x} = \mathbf{0}$, and write

$$\begin{pmatrix} h_1 \\ h_2 \\ \dots \\ h_m \end{pmatrix} = A\mathbf{x}$$

Then we have

$$A^TA\mathbf{x} = \mathbf{0} \quad \Rightarrow \quad \mathbf{x}^TA^TA\mathbf{x} = \mathbf{0}$$

by left multiplication by $\mathbf{x}^T$ on both sides. This means that

$$\mathbf{x}^T A^T A\mathbf{x} = (A\mathbf{x})^T (A\mathbf{x}) = 0 \quad \Rightarrow \quad h_1^2 + h_2^2 + \cdots + h_m^2 = 0$$

The last equation implies that $h_1 = h_2 = \cdots = h_m = 0$; that is, that $A\mathbf{x} = \mathbf{0}$. This means that any solution of $A^T A\mathbf{x} = \mathbf{0}$ is also a solution of $A\mathbf{x} = \mathbf{0}$. Hence the two solution sets are equal, and this proves a). To prove b), consider the two linear systems $A\mathbf{x} = \mathbf{0}$ and $\mathbf{A}^T A\mathbf{x} = \mathbf{0}$ in n variables. Since the two linear systems have the same solutions, they must have the same number d of free variables. This gives

$$\text{rk}(A) = n - d, \quad \text{rk}(A^T A) = n - d$$

so $\text{rk}(A) = \text{rk}(A^T A)$.

13 Midterm Exam in GRA6035 24/09/2010, Problem 2
We compute the determinant

$$\begin{vmatrix} 2 & 1 & 0 \\ 3 & 2 & 1 \\ -1 & 1 & h \end{vmatrix} = h - 3$$

Hence the vectors are linearly independent exactly when $h \neq 3$, and the correct answer is alternative **D**. This question can also be answered using Gauss elimination.

14 Mock Midterm Exam in GRA6035 09/2010, Problem 1
The vector $\mathbf{w}$ is a linear combination of the vectors in $\mathscr{B}$ if and only if the linear system

$$x_1 \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} + x_2 \begin{pmatrix} 5 \\ -4 \\ -7 \end{pmatrix} + x_3 \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \\ h \end{pmatrix}$$

is consistent. We write down the augmented matrix of the system and reduce it to echelon form

$$\begin{pmatrix} 1 & 5 & -3 & -4 \\ -1 & -4 & 1 & 3 \\ -2 & -7 & 0 & h \end{pmatrix} \dashrightarrow \begin{pmatrix} 1 & 5 & -3 & -4 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & h-5 \end{pmatrix}$$

The system is consistent if and only if $h = 5$. Hence the correct answer is alternative **C**. This question can also be answered using minors.

15 Midterm Exam in GRA6035 24/05/2011, Problem 3
We compute the determinant

$$\begin{vmatrix} 2 & 1 & h+1 \\ 3 & 2 & h \\ -1 & 1 & h-2 \end{vmatrix} = 3h + 3$$

Hence the vectors are linearly independent exactly when $h \neq -1$, and the correct answer is alternative **D**. This question can also be answered using Gauss elimination.