Problem Session 3
GRA 6035 Mathematics

November 12, 2012

BI Norwegian Business School

Problems

1. Write down the Kuhn-Tucker conditions and solve them in the following Kuhn-Tucker problems:

a) $\max f(x,y) = xy$ subject to $x + 4y \leq 16$
b) $\max f(x,y) = x^2y$ subject to $2x^2 + y^2 \leq 3$
c) $\max f(x,y,z) = xyz$ subject to $x^2 + y^2 \leq 1$ and $x + z \geq 1$
d) $\max f(x,y) = xy$ subject to $x^2 + y^2 \leq 1$
e) $\max f(x,y,z) = xyz$ subject to $x + y + z \leq 1$, $x \geq 0$, $y \geq 0$ and $z \geq 0$
f) $\max f(x,y,z) = xyz + z$ subject to $x^2 + y^2 + z \leq 6$, $x \geq 0$, $y \geq 0$ and $z \geq 0$

2. Solve the Kuhn-Tucker problems in Problem 1.

3. When x thousand dollars is spent on labor and y thousand dollars is spent on equipment, a certain factory produces $Q(x,y) = 50x^{1/2}y^2$ units of output.

a) How should \$80.000 be allocated between labor and equipment to yield the largest possible output?
b) Use an envelope theorem to estimate the change in maximum output if this allocation is decreased by \$1.000.
c) Compute the exact change in b).

4. Write down the Lagrange conditions and solve them in the following Lagrange problems:

a) $\max f(x,y) = xy$ subject to $x + 4y = 16$
b) $\max f(x,y) = x^2y$ subject to $2x^2 + y^2 = 3$
c) $\max f(x,y,z) = xyz$ subject to $x^2 + y^2 = 1$ and $x + z = 1$
d) $\min f(x,y) = x^2 + y^2$ subject to $x^2 + xy + y^2 = 3$
e) $\min f(x,y,z) = x^2 + y^2 + z^2$ subject to $3x + y + z = 5$ and $x + y + z = 1$
f) $\max / \min f(x,y,z) = x + y + z^2$ subject to $x^2 + y^2 + z^1 = 1$ and $y = 0$
g) $\max f(x,y,z) = xz + yz$ subject to $y^2 + z^2 = 1$ and $xz = 3$
h) $\max x^2y^2z^2$ subject to $x^2 + y^2 + z^2 = 3$

5. Solve the Lagrange problems in Problem 4.