BI HANDELSHØYSKOLEN

Nydalsveien 37, 0484 Oslo
Postadresse 0442 Oslo

Telefon 06600
Telefaks 21 04 80 00
Studieinfo. tlf. 810 00 500

www.bi.no
info@bi.no
Org. nr. 971 22 8865

JUSTIFICATION OF GRADES

Grading scale:

A	92%
B	77%
C	58%
D	46%
E	40%

$100\% = 12 \cdot 6 = 72p$
(no extra credit)

Max score: 108%

Average score: $39.1p/72p = 54\%$ D

1 a) 5,5p
b) 4,1p
c) 4,0p

2. a) 3,5p
b) 2,5p
c) 2,7p

3. a) 4,4p
b) 3,2p.
c) 0,9p.

4. a) 4,1p
b) 2,0p.
c) 1.3p.

5. 1.1p

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1. $A = \begin{pmatrix} -2 & 2 & 0 \\ -1 & 0 & 2 \\ 0 & -1 & 2 \end{pmatrix}$

a) $|A| = \_\_\_\_ = 0$ 3p.

$rk\, A =$ Gauss / minors 3p.

b) $A \cdot \underline{x} = \underline{0}$

$$A = \begin{pmatrix} -2 & 2 & 0 \\ -1 & 0 & 2 \\ 0 & -1 & 2 \end{pmatrix} \to \begin{pmatrix} -1 & 0 & 2 \\ 0 & 2 & -4 \\ 0 & -1 & 2 \end{pmatrix} \to \begin{pmatrix} -1 & 0 & 2 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$x = 2z$
$y = 2z$
z fri

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = z \cdot \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$ 5p.

Span of

$\underline{v}_1 = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$ 1p.

c) $\begin{vmatrix} -2-\lambda & 2 & 0 \\ -1 & -\lambda & 2 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0$ $\quad (-2-\lambda)(-\lambda(2-\lambda)+2) - 2(-1)(2-\lambda) = 0$

2p.

$\underline{-\lambda^3 = 0}$ $\quad \lambda_1 = \lambda_2 = \lambda_3 = 0$

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2.

a) $y_{t+2} = 3y_{t+1} - 2y_t \qquad y_0 = 1,\ y_1 = 2$

$y_{t+2} - 3y_{t+1} + 2y_t = 0$

$r^2 - 3r + 2 = 0 \rightarrow y_t = \underline{C_1 \cdot 1^t + C_2 \cdot 2^t}$ 4p.

$r = 1, 2$

$y_0 = 1: \quad C_1 \cdot 1^0 + C_2 \cdot 2^0 = C_1 + C_2 = 1 \qquad C_1 = 0$

$y_1 = 2: \quad C_1 \cdot 1^1 + C_2 \cdot 2^1 = C_1 + 2C_2 = 2 \qquad C_2 = 1$

$\underline{\underline{y_t = 2^t}}$

b) $y' - y\ln t = y \rightarrow y' - y\ln t - y = 0$

$y' - y \cdot (\ln t + 1) = 0$ linear

$u = e^{\int -(\ln t + 1)dt}$ ← int. factor

3p.

$y' = y + y\ln t$

$= y \cdot (1 + \ln t)$

separable 3p.

Computation 3p.

$\int \frac{1}{y} y' dt = \int 1 + \ln t \, dt$

c) $y e^{ty} + t e^{ty} y' = 1, \qquad y(1) = \ln(2)$

$\underbrace{(y e^{ty} - 1)}_{h'_t} + \underbrace{(t e^{ty})}_{h'_y} y' = 0 \longrightarrow h = C$

$h = \underline{e^{ty} - t = C}$ 4p. $\rightarrow$

$e^{ty} = t + C$

$ty = \ln(t + C)$

$y = \frac{\ln(t+C)}{t}, \quad C = 1$

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3. a)

$$H(f) = \begin{pmatrix} 10 & -8 & -4 \\ -8 & 10 & -4 \\ -4 & -4 & 16 \end{pmatrix} \qquad A = \begin{pmatrix} 5 & -4 & -2 \\ -4 & 5 & -2 \\ -2 & -2 & 8 \end{pmatrix}$$

ok full score

$D_1 = 10$ $\quad \Delta_1 = 10, 10, 16$

$D_2 = 36$ $\quad \Delta_2 = 36, 144, 144$

$D_3 = 0$ $\quad \Delta_3 = 0$

$\Bigg\}$ f convex

b)

$$f'_x = 10x - 8y - 4z = 0$$
$$f'_y = -8x + 10y - 4z = 0$$
$$f'_z = -4x - 4y + 16z = 0$$

$\Downarrow$

$x = 2z$

$y = 2z$

z is free

$x = -y + 4z$

$-8\cdot(-y+4z) + 10y - 4z = 0$

$10\,(-y+4z) - 8y - 4z = 0$

$\Downarrow$

$0 \cdot z = 0 \;\not\Rightarrow\; z = 0$ 4p.

Inf. many stationary pts:

$(2z, 2z, z)$

c)

$g(w) = w \cdot \ln(w)$, $\quad w = f(x,y,z)$

$g'(w) = 1 \cdot \ln(w) + w \cdot \frac{1}{w}$

$= \ln(w) + 1 \geq 1$

g increasing

$w = f(x,y,z)$ convex

$\Downarrow$

$(0,0,0)$ global min for f

$w \geq 1 \Leftarrow f(0,0,0) = 1$

Min of g ~~min~~ at $(0,0,0)$

$= g(f(0,0,0)) = g(1) = 0$

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4.

a) $\mathcal{L} = f(x,y,z) - \lambda \cdot g(x,y,z)$
$= 5x^2 + \ldots + 8z^2 + 1 - \lambda \cdot (x+y-4z)$

Lagrange condi: FOC + C

$$\boxed{\begin{aligned} \mathcal{L}'_x &= 10x - 8y - 4z - \lambda = 0 \\ \mathcal{L}'_y &= -8x + 10y - 4z - \lambda = 0 \\ \mathcal{L}'_z &= -4x - 4y + 16z + 4\lambda = 0 \\ x + y - 4z &= 8 \end{aligned}}$$

4p.

$10x - 8y - 4z = \lambda$
$-8x + 10y - 4z = \lambda$
$-4x - 4y + 16z = -4z$
$x + y - 4z = 8$

ok

Linear system in x, y, z, λ

$$\left(\begin{array}{cccc|c} 10 & -8 & -4 & -1 & 0 \\ -8 & -4 & -4 & -1 & 0 \\ -4 & -4 & 16 & 4 & 0 \\ 1 & 1 & -4 & 0 & 8 \end{array}\right)$$

$\Big\downarrow$ Gauss

b)

$$\left(\begin{array}{cccc|c} 1 & 1 & -4 & 0 & 8 \\ 0 & -18 & 36 & -1 & -80 \\ 0 & 0 & 0 & -2 & -16 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$x = -y + 4z + 8$
$= -(2z+4) + 4z + 8 = \underline{2z+4}$

$y = \underline{2z+4}$

$\Uparrow$

$-18y = -36\cancel{z} + \lambda - 80$

$\lambda = 8$

z free

Cand pts: $(2z+4, 2z+4, z; 8)$ for all z. 3p

SOC: $\mathcal{L}(x,y,z;8) = f(x,y,z) - 8(x+y-4z)$

f convex $\Rightarrow$ $\mathcal{L}(x,y,z;8)$ convex

$\Rightarrow$ $(2z+4, 2z+4, z)$ are global min

3p

Min value: $f(4,4,0) = 33$

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c) min $f(x,y,z)$ when $x+y-4z=\underline{a}$

$\Updownarrow$

$\mathcal{L} = f(x,y,z) - \lambda \cdot (x+y-4z-a)$

3p.

Env. thm: $\dfrac{df^*(a)}{da} = \dfrac{\partial \mathcal{L}}{\partial a}\Big|_{(x,y,z,\lambda)=(x^*(a),y^*(a),z^*(a);\lambda^*(a))}$

$= \lambda^*(a)$

when $(x^*(a), y^*(a), z^*(a); \lambda^*(a))$ is a ~~...~~ solution which solves FOC+C.

$$f^*(7.92) \simeq \underbrace{f^*(8)}_{=33} + \underbrace{\lambda^*(8)}_{=8} \cdot \underbrace{\Delta a}_{=(-0.08)} = 33 - 0.64 = \underline{\underline{32.36}}$$

3p.

5. $A = \begin{pmatrix} -\alpha_2 & \alpha_1 & 0 \\ -\alpha_3 & 0 & \alpha_1 \\ 0 & -\alpha_3 & \alpha_2 \end{pmatrix}$

$|A| = -\alpha_2(\alpha_1\alpha_3) - \alpha_1 \cdot (-\alpha_3\alpha_2)$

$= -\alpha_1\alpha_2\alpha_3 + \alpha_1\alpha_2\alpha_3 = \underline{0}$

$\underline{\underline{\operatorname{rk} A \le 2}}$ 2p.

$\alpha_1\alpha_3$ $\quad \alpha_1^2 \quad -\alpha_2^2 \quad \alpha_3^2$

$\operatorname{rk} A < 2 \iff$ all 2-minors are 0

$$\left.\begin{array}{l} \alpha_1^2 = 0 \\ -\alpha_2^2 = 0 \\ \alpha_3^2 = 0 \end{array}\right\} \Rightarrow \begin{array}{l} \alpha_1 = 0 \\ \alpha_2 = 0 \\ \alpha_3 = 0 \end{array}$$

$$\operatorname{rk} A = \begin{cases} 2, & (\alpha_1,\alpha_2,\alpha_3) \neq (0,0,0) \\ 0, & \quad " \quad = (0,0,0) \end{cases}$$