

GRA 6035 MATHEMATICS

Problems for Lecture 7

Key problems

Problem 1.

Check if the sets given by these conditions are convex and compact (closed and bounded). It can be useful to sketch the sets:

a) $x, y \geq 0$ and $2x + 3y \leq 6$ b) $4x^2 + 9y^2 \leq 36$ c) $x, y \geq 1$ and $2x + 3y \geq 12$ d) $xyz \leq 1$ and $x, y, z > 0$

Problem 2.

Determine whether the functions $f(x, y) = |x - y|$ and $g(x, y, z) = 1 - e^{x-y+z}$ are convex or concave.

Problem 3.

Solve the Lagrange problems. You may assume that all admissible points satisfy the NDCQ:

a) $\max f(x, y, z) = x + 2y + 3z$ when $2x^2 + y^2 + 2z^2 = 9$ b) $\max / \min f(x, y, z) = x^4 + y^4 + z^4$ when $2x^2 + y^2 + 2z^2 = 9$

Problem 4.

Solve the Kuhn-Tucker problems. You may assume that all admissible points satisfy the NDCQ:

a) $\max f(x, y, z) = x - 2y + z$ when $x^2 + y^2 + z^2 \leq 3$ b) $\max f(x, y, z) = \ln(xyz)$ when $2x^2 + y^2 + 2z^2 \leq 6$

Problems from the Digital Workbook

Exercise problems 7.1 - 7.10 (full solutions in the workbook)
Exam problems 7.11 (full solutions in the workbook)

Answers to key problems

Problem 1.

a) Convex and compact set b) Convex and compact set c) Convex but not compact set (not bounded) d) Not convex and not compact set (not bounded)

Problem 2.

The function f is convex, but not concave. The function g is concave, but not convex.

Problem 3.

a) $f_{\max} = 9$ b) $f_{\max} = 81, f_{\min} = 9$

Problem 4.

a) $f_{\max} = 3\sqrt{2}$ b) $f_{\max} = \ln(2)/2$