An Example

Let $f(x,y) = x^2y^3 + y^2 - 2y$. Then f has just <u>one</u> stationary pt, which is a <u>local min</u>. However, f has <u>no global min</u>.

$$\left.\begin{aligned} f'_x &= 2xy^3 = 0 \\ f'_y &= 3x^2y^2 + 2y - 2 = 0 \end{aligned}\right\}$$

FOC: $x = 0$ or $y = 0$

If $x = 0$, then (2) gives

$2y - 2 = 0 \Rightarrow \underline{y = 1}$

If $y = 0$, then (2) gives

$-2 = 0$ <u>no solution</u>

$$H(f) = \begin{pmatrix} 2y^3 & 6xy^2 \\ 6xy^2 & 6x^2y + 2 \end{pmatrix}$$

$$H(f)(0,1) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \qquad \begin{aligned} D_1 &= 2 > 0 \\ D_2 &= 4 > 0 \end{aligned}$$

pos. defn. $\Rightarrow$ $(0,1)$ <u>local min</u>

<u>Conclusion</u>: $(0,1)$ is the only stationary pt, with $\underline{f(0,1) = -1}$

But $(x,y) = (0,1)$ with $f(0,1) = -1$ is <u>not</u> global min

For example,

$$f(3,-1) = -9 + 1 + 2 = -6 < -1$$

<u>Conclusion</u>: f has no global min even if $(0,1)$ is local min and this is the only stationary point.