Exam	Final exam in GRA 6035 Mathematics
Date	March 4th, 2016 at 1500 - 1800

The exam consists of 12 problems that have the same weight and give maximal score 6p each, giving a maximal score of 72p (100%). In adddition, there is one additional problem for 6p extra credits (can be skipped).

Question 1.

We consider the matrix A given by

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

(a) **(6p)** Compute the rank of A and find all the solutions of the linear system $A \cdot \mathbf{x} = \mathbf{0}$.
(b) **(6p)** Is A diagonalizable? Find all eigenvalues of A and their multiplicities.
(c) **(6p)** Write down the quadratic form $Q(x, y, z, w)$ with symmetric matrix A, and determine its definiteness.

We consider a Markov chain given by $\mathbf{x}_{t+1} = T\mathbf{x}_t$, where the transition matrix T is given by

$$T = \begin{pmatrix} 0.75 & 0.25 & 0.10 \\ 0.10 & 0.60 & 0.05 \\ 0.15 & 0.15 & 0.85 \end{pmatrix}$$

and the inital state is $\mathbf{x}_0$.

(d) **(6p)** Find the equilibrium state $\mathbf{x} = \lim_{t \to \infty} T^t \, \mathbf{x}_0$

Question 2.

Solve the differential equations:

(a) **(6p)** $y'' - 16y = e^{-t}$
(b) **(6p)** $(3t^2y + 2ty^2 + t^3) + (t^3 + 2yt^2)y' = 0$
(c) **(6p)** $y' = \frac{yt}{\ln(y)}$ with initial condition $y(0) = e$

Question 3.

We consider the function given by $f(x, y, z) = e^{1-u}$, where $u = u(x, y, z)$ is the quadratic form

$$u(x, y, z) = x^2 + 2xy + 3y^2 + 2yz + z^2$$

(a) **(6p)** Compute the partial derivatives of f, and find all its stationary points.
(b) **(6p)** Classify all stationary points of f as local maxima, local minima or saddle points.
(c) **Extra credits (6p)** Does f have a global maximum or minimum? Justify your answer.

Question 4.

We consider the following Kuhn-Tucker problem:

$$\max\ f(x,y) = xy \text{ subject to } \begin{cases} 4x^2 + 9y^2 \le 36 \\ 2x + 3y \ge 6 \end{cases}$$

(a) **(6p)** Sketch the set of admissible points. Is it bounded?
(b) **(6p)** Find all points $(x, y; \lambda_1, \lambda_2)$ with $\lambda_1 = 1/12$ that satisfy the Kuhn-Tucker conditions.
(c) **(6p)** Solve the Kuhn-Tucker problem, and find the maximum value if it exists.