Exam	Final exam in GRA 6035 Mathematics
Date	January 6th, 2017 at 1200 - 1500

All subquestions have the same weight and give maximal score 6p each. Answers to the first 12 subquestions give a maximal score of 72p (100%). Question 5 can be skipped, but gives 6p extra credit if answered correctly.

Question 1.

We consider the matrix A given by

$$A = \begin{pmatrix} -1 & 1 & -1 \\ -1 & 0 & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

(a) **(6p)** Compute the determinant and rank of A.
(b) **(6p)** Solve the linear system $A \cdot \mathbf{x} = \mathbf{0}$, and write the solutions in the form $\text{span}(\mathbf{v}_1, \dots, \mathbf{v}_r)$.
(c) **(6p)** Is A diagonalizable?

Question 2.

Solve the differential equations:

(a) **(6p)** $y'' - 3y' - 10y = 20$
(b) **(6p)** $y' + \ln(t) = y \ln(t)$
(c) **(6p)** $6t(y^2 - t^2)^2 = 6y(y^2 - t^2)^2 \cdot y', \quad y(0) = 1$

Question 3.

We consider the function given by $f(x, y, z) = -3 - 2x^2 + 2xy - 2xz - 2y^2 + 4yz - 2z^2$.

(a) **(6p)** Is f convex? Is f concave?
(b) **(6p)** Find the maximum value of f, if it exists.

We consider the function $g(x, y, z) = 6/w$ with $w = f(x, y, z)$.

(c) **(6p)** Find the maximum and minimum value of g, if they exist.

Question 4.

We consider the following Kuhn-Tucker problem:

$$\max \; f(x, y, z) = -3 - 2x^2 + 2xy - 2xz - 2y^2 + 4yz - 2z^2 \text{ subject to } x + y - z \geq 2$$

(a) **(6p)** Write down all Kuhn-Tucker conditions for this problem.
(b) **(6p)** Solve the Kuhn-Tucker problem. What is the maximum value?
(c) **(6p)** Consider the Kuhn-Tucker problem where the constraint is replaced by $1.12x + y - z \geq 2$. State the relevant envelope theorem, and use it to estimate the new maximum value.

Question 5.

Let $\alpha_1, \alpha_2, \alpha_3$ be parameters, and consider the matrix A given by

$$A = \begin{pmatrix} -\alpha_2 & \alpha_1 & 0 \\ -\alpha_3 & 0 & \alpha_1 \\ 0 & -\alpha_3 & \alpha_2 \end{pmatrix}$$

Extra credits (6p)

Find the eigenvalues of A. For which values $(\alpha_1, \alpha_2, \alpha_3)$ is A diagonalizable?