NORWEGIAN BUSINESS SCHOOL

Solutions:	GRA 60352	Mathematics		
Examination date:	30.04.2014	09:00 – 10:00	Total no. of pages:	3
			No. of attachments:	0
Permitted examination	A bilingual dictionary and BI-approved calculator TEXAS			
support material:	INSTRUMENTS BA II Plus			
Answer sheets:	Answer sheet for multiple-choice examinations			
	Counts 20% of GRA 6035		The questions have equal weight	
Re-take exam			Responsible department: Economics	

Correct answers: C-D-C-A-B-C-A-B

Question 1.

The linear system is consistent since it is homogeneous. It has $n - \text{rk}\, A = 3 - 2 = 1$ degrees of freedom. The correct answer is alternative **C**.

Question 2.

We form the matrix with the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ as columns, and compute its determinant

$$\begin{vmatrix} 0 & 1 & s \\ 1 & -s & s \\ 1 & s & 1 \end{vmatrix} = 1 \cdot (s + s^2) - 1 \cdot (1 - s^2) = 2s^2 + s - 1$$

We have $2s^2 + s - 1 = 0$ when $s = -1$ or $s = 1/2$. This shows that the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly independent when $s \neq -1, 1/2$, and linearly dependent if $s = -1$ or $s = 1/2$. The correct answer is alternative **D**.

Question 3.

We reduce the matrix A to an echelon form:

$$\begin{pmatrix} 1 & 1 & -1 & -1 \\ 2 & 3 & t & t-2 \\ 1 & 3 & 3 & -1 \end{pmatrix} \dashrightarrow \begin{pmatrix} 1 & 1 & -1 & -1 \\ 0 & 1 & t+2 & t \\ 0 & 2 & 4 & 0 \end{pmatrix} \dashrightarrow \begin{pmatrix} 1 & 1 & -1 & -1 \\ 0 & 1 & t+2 & t \\ 0 & 0 & -2t & -2t \end{pmatrix}$$

There will be a pivot in the third row when $t \neq 0$, and no pivot when $t = 0$. The correct answer is alternative **C**.

Question 4.

The characteristic equation of A is

$$\begin{vmatrix} 3-\lambda & -2 & 1 \\ 0 & 2-\lambda & -3 \\ 0 & 0 & 4-\lambda \end{vmatrix} = (3-\lambda)(2-\lambda)(4-\lambda) = 0$$

Hence the eigenvalues of A are $\lambda = 3$, $\lambda = 2$ and $\lambda = 4$. The correct answer is alternative **A**.

Question 5.

We compute $A\mathbf{v}$ and compare with $\lambda\mathbf{v}$, and see that

$$\begin{pmatrix} 2 & 3 \\ t & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 2t-1 \end{pmatrix}$$

is a multiple of $\mathbf{v}$ if and only if $t = 9/4$ (with $\lambda = 7/2$). The correct answer is alternative **B**.

Question 6.

The symmetric matrix of the quadratic form $f(x_1, x_2, x_3, x_4) = x_1^2 + 3x_1x_4 + 2x_2^2 - 8x_2x_4 + 3x_3^2 + 7x_4^2$ is given by

$$A = \begin{pmatrix} 1 & 0 & 0 & 3/2 \\ 0 & 2 & 0 & -4 \\ 0 & 0 & 3 & 0 \\ 3/2 & -4 & 0 & 7 \end{pmatrix}$$

The leading principal minors are $D_1 = 1$, $D_2 = 1 \cdot 2 = 2$, $D_3 = 1 \cdot 2 \cdot 3 = 6$, and $D_4 = |A|$ is given by

$$D_4 = \begin{vmatrix} 1 & 0 & 0 & 3/2 \\ 0 & 2 & 0 & -4 \\ 0 & 0 & 3 & 0 \\ 3/2 & -4 & 0 & 7 \end{vmatrix} = 3(1(14-16) + 3/2(0-3)) = -39/2$$

Hence f is indefinite. The correct answer is alternative **C**.

Question 7.

We compute the first order derivatives to find stationary points, and find

$$3x^2 + 3y^2 - 3 = 0, \quad 6xy = 0, \quad -4z^3 + 4 = 0$$

This gives four stationary points $(\pm 1, 0, 1)$ and $(0, \pm 1, 1)$. We compute the Hessian matrix of f and find

$$H(f) = \begin{pmatrix} 6x & 6y & 0 \\ 6y & 6x & 0 \\ 0 & 0 & -12z^2 \end{pmatrix} = \begin{pmatrix} \pm 6 & 0 & 0 \\ 0 & \pm 6 & 0 \\ 0 & 0 & -12 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 0 & \pm 6 & 0 \\ \pm 6 & 0 & 0 \\ 0 & 0 & -12 \end{pmatrix}$$

Since $D_2 = -36$ at the last two stationary points, they are saddle points. At $(1, 0, 1)$ we have $D_1 = 6$, $D_2 = 36$ and $D_3 = -12 \cdot 36 < 0$ so this is also a saddle point. At $(-1, 0, 1)$ we have $D_1 = -6$, $D_2 = 36$ and $D_3 = -12 \cdot 36 < 0$ so this is a local maximum. It follows that there are local max but not local min for f. The correct answer is alternative **A**.

Question 8.

The function $f(x,y,z) = x^{-2}\, e^{ay}$ has Hessian matrix

$$H(f) = \begin{pmatrix} 6x^{-4}e^{ay} & -2ax^{-3}e^{ay} \\ -2ax^{-3}e^{ay} & a^2x^{-2}e^{ay} \end{pmatrix}$$

Hence $D_1 = 6x^{-4}e^{ay} > 0$ and $D_2 = 2a^2x^{-6}e^{2ay} > 0$. It follows that $H(f)$ is positive definite for all (x,y) with $x > 0$. Hence f is convex for all a. The correct answer is alternative **B**.