Solutions:	**GRA 60352**	**Mathematics**		
Examination date:	07.03.2016	18:00 – 19:00	Total no. of pages:	2
			No. of attachments:	0
Permitted examination support material:	A bilingual dictionary and BI-approved calculator			
Answer sheets:	Answer sheet for multiple-choice examinations			
	Counts 20% of GRA 6035		The questions have equal weight	
Re-take exam			Responsible department: Economics	

Correct answers: C-C-D-C-A-D-A-B

Question 1.

The linear system is consistent with a one degree of freedom since it has rank 3 (that is, a pivot in three of the first four columns). The correct answer is alternative **C**.

Question 2.

We form the matrix with the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ as columns, and compute its determinant

$$\begin{vmatrix} 1 & 3 & 5 \\ -1 & 2 & 0 \\ s & s+1 & s+3 \end{vmatrix} = 5(-1(s+1) - 2s) + (s+3)(2+3) = 5(-3s-1) + 5(s+3) = 10 - 10s$$

This shows that the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly independent when $s \neq 1$, and linearly dependent if $s = 1$. The correct answer is alternative **C**.

Question 3.

We use elementary row operations to find an echelon form:

$$\begin{vmatrix} 1 & 4 & -7 & 3 \\ 2 & -2 & 8 & 0 \\ 2 & 10 & t-16 & 1-t \end{vmatrix} \to \begin{vmatrix} 1 & 4 & -7 & 3 \\ 0 & -10 & 22 & -6 \\ 0 & 2 & t-2 & -t-5 \end{vmatrix} \to \begin{vmatrix} 1 & 4 & -7 & 3 \\ 0 & 2 & t-2 & -t-5 \\ 0 & 0 & 5t+12 & -5t-31 \end{vmatrix}$$

It follows that the rank of A is 3 for all values of t since the last row will have a pivot for all values of t. The correct answer is alternative **D**.

Question 4.

The characteristic equation of A is

$$\begin{vmatrix} 1-\lambda & 0 & 2 \\ 0 & -2-\lambda & 0 \\ 1 & 0 & -3-\lambda \end{vmatrix} = (-2-\lambda)(\lambda^2+2\lambda-5) = 0$$

Hence the eigenvalues of A are $\lambda_1 = -2$ and λ_2, λ_3 such that $\lambda_2 + \lambda_3 = -2,\ \lambda_2\lambda_3 = -5$. Since $-5 < 0$, exactly one of the eigenvalues λ_2, λ_3 are negative. The correct answer is alternative **C**.

Question 5.

The eigenvalues are the numbers $1, 2, 3$ on the diagonal since A is upper triangular. The matrix is diagonalizable since it has three distinct eigenvalues. The correct answer is alternative **A**.

Question 6.

Eigenvectors for $\lambda = 1$ are given by the linear system $(A - I)\mathbf{x} = \mathbf{0}$, where

$$A - I = \begin{pmatrix} -0.35 & 0.21 \\ 0.35 & -0.21 \end{pmatrix}$$

Therefore, we see that $x = 3$ and $y = 5$ gives one eigenvector, and all others are multiple of this one. Multiplication by $1/8$ gives the state vector with $x = 3/8$ and $y = 5/8$. The correct answer is alternative **D**.

Question 7.

The Hessian matrix of $f(x, y) = \ln(xy - 1)$ is given by

$$H(f) = \frac{1}{(xy-1)^2} \cdot \begin{pmatrix} -y^2 & -1 \\ -1 & -x^2 \end{pmatrix}$$

The leading principal minors are $D_1 = -y^2/(xy-1)^2$ and $D_2 = (x^2y^2 - 1)/(xy-1)^4$. Since $x, y > 0$ and $xy > 1$, we have that $D_1 < 0$ and $D_2 > 0$ for all (x, y) in D_f. Hence $H(f)$ is negative definite at all points and f is concave (but not convex). The correct answer is alternative **A**.

Question 8.

The function $f(x, y, z) = 12 - 25x^{1/5}y^{3/5}$ has Hessian matrix

$$H(f) = \begin{pmatrix} 4x^{-9/5}y^{3/5} & -3x^{-4/5}y^{-2/5} \\ -3x^{-4/5}y^{-2/5} & 6x^{1/5}y^{-7/5} \end{pmatrix}$$

Hence $D_1 = 4x^{-9/5}y^{3/5}$ and $D_2 = 15x^{-8/5}y^{-4/5}$. Since $x, y > 0$, this implies that $D_1, D_2 > 0$ for all points (x, y) in D_f. This means that $H(f)$ is positive definite at all points and f is convex (but not concave). The correct answer is alternative **B**.