BI NORWEGIAN SCHOOL OF MANAGEMENT

Department of Economics

Solutions to mock exam:	**GRA 60352 Mathematics**
Examination date:	17.09.2010, 14:00 – 15:00
Permitted examination aids:	Bilingual dictionary.
	BI-approved exam calculator: TEXAS INSTRUMENTS BA II Plus^{TM}
Answer sheets:	Answer sheet for multiple choice examinations
Total number of pages:	4
Number of attachments:	1 (example of how to use the answer sheet)

PLEASE READ THE FOLLOWING BEFORE YOU BEGIN!

- Students must themselves assure that the examination papers are complete.
- Students must provide the following information on the answer sheet:
  - Examination code
  - Personal initials
  - ID number

  The student registration number must be recorded with both the appropriate numbers and by putting an "X" by the corresponding number in the columns below.
- Pens with green ink and pencils cannot be used in filling in answer sheets. Answer sheets must not be used for writing rough drafts.
- **All answers must be recorded with an "X" under the letter you believe corresponds with the correct answer.**
- **Cancel an "X" by filling in the box completely (boxes that are completely filled in will not be registered). "X" in two boxes for one question will be registered as a wrong answer.**
- The attached example shows you how the answer sheet would be filled in if A were the correct answer for question 1, B correct for question 2, C correct for question 3 and D correct for question 4. An "X" under E indicates that you choose not to answer question 5.
- **Your answers are to be recorded on the answer sheet. Answers written on the examination papers and not on the answer sheets will not be graded.**
- There is only <u>one</u> right answer for each question. Because the questions are weighted equally, it can be to your advantage to answer the simplest questions first.
- Wrong answers are given -1 point, unanswered questions get 0 points (indicated by an "X" next to E") and correct answers are given 3 points.
- You can keep the examination papers.

Good luck!

Correct answers: D-C-C-C-C-C-D-D

Question 1.

Since the augmented matrix of the system is in echelon form, we see that the system is inconsistent. Hence the correct answer is alternative $\boxed{D}$. This question can also be answered using minors.

Question 2.

The vector $\mathbf{w}$ is a linear combination of the vectors in $\mathcal{B}$ if and only if the linear system

$$x_1\begin{pmatrix}1\\-1\\-2\end{pmatrix}+x_2\begin{pmatrix}5\\-4\\-7\end{pmatrix}+x_3\begin{pmatrix}-3\\1\\0\end{pmatrix}=\begin{pmatrix}-4\\3\\h\end{pmatrix}$$

is consistent. We write down the augmented matrix of the system and reduce it to echelon form

$$\begin{pmatrix}1&5&-3&-4\\-1&-4&1&3\\-2&-7&0&h\end{pmatrix}\dashrightarrow\begin{pmatrix}1&5&-3&-4\\0&1&-2&-1\\0&0&0&h-5\end{pmatrix}$$

The system is consistent if and only if $h=5$. Hence the correct answer is alternative $\boxed{C}$. This question can also be answered using minors.

Question 3.

We compute an echelon form of A using elementary row operations, and get

$$A=\begin{pmatrix}1&2&-5&0&-1\\2&5&-8&4&3\\-3&-9&9&-7&-2\\3&10&-7&11&7\end{pmatrix}\dashrightarrow\begin{pmatrix}1&2&-5&0&-1\\0&1&2&4&5\\0&0&0&5&10\\0&0&0&0&0\end{pmatrix}$$

Hence A has rank 3, and the correct answer is alternative $\boxed{C}$. This answer can also be answered using minors. For instance, the minor of order 3 obtained by deleting row 4 and column 3 and 5 is non-zero.

Question 4.

The characteristic equation of A is $\lambda^2-10\lambda+25=0$. Hence the eigenvalues of A is $\lambda=5$ (double root), and the correct answer is alternative $\boxed{C}$.

Question 5.

The characteristic equation of A is

$$(\lambda^2-6\lambda+9)(-1-\lambda)=0$$

which gives eigenvalues $\lambda=3$ (double root) and $\lambda=-1$. We compute the eigenspace for $\lambda=3$, and find that it has only one degree of freedom. Hence A is not diagonalizable, and the correct answer is alternative $\boxed{C}$.

Question 6.

Since all terms of f have degree two, it is a quadratic form, and its symmetric matrix is

$$A=\begin{pmatrix}1&3&0\\3&3&0\\0&0&2\end{pmatrix}$$

The characteristic equation of A is $(\lambda^2-4\lambda-6)(2-\lambda)=0$, and the eigenvalues are $\lambda=2$ and $\lambda=2\pm\sqrt{10}$. Hence the correct answer is alternative $\boxed{C}$.

Question 7.

The function f is a sum of a constant function and the quadratic form $-aQ(x_1, x_2)$. Since Q is positive definite, it is convex, and $-Q$ is concave. If $a \geq 0$, then $-aQ(x_1, x_2) = a(-Q(x_1, x_2))$ is concave. If $a \leq 0$, then $-a \geq 0$ and $-aQ(x_1, x_2)$ is convex. The correct answer is alternative $\boxed{D}$.

Question 8.

The system is consistent since $\mathbf{x} = \mathbf{0}$ is a solution. The rank of A depends on the coefficients of A, but

$$\operatorname{rk} A \leq 57$$

since A has 57 rows. Moreover, $n - \operatorname{rk} A \geq 61 - 57 = 4$. Hence the system has at least four degrees of freedom, and the correct answer is therefore alternative $\boxed{D}$.