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Mock exam in:	<b>GRA 60352 Mathematics</b>
Examination date:	17.09.2010, 14:00 – 15:00
Permitted examination aids:	Bilingual dictionary.  BI-approved exam calculator: TEXAS INSTRUMENTS BA II Plus™
Answer sheets:	Answer sheet for multiple choice examinations
Total number of pages:	4
Number of attachments:	1 (example of how to use the answer sheet)

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**PLEASE READ THE FOLLOWING BEFORE YOU BEGIN!**

- Students must themselves assure that the examination papers are complete.
- Students must provide the following information on the answer sheet:
  - Examination code
  - Personal initials
  - ID number

The student registration number must be recorded with both the appropriate numbers and by putting an “X” by the corresponding number in the columns below.

- Pens with green ink and pencils cannot be used in filling in answer sheets. Answer sheets must not be used for writing rough drafts.
- **All answers must be recorded with an “X” under the letter you believe corresponds with the correct answer.**
- **Cancel an “X” by filling in the box completely (boxes that are completely filled in will not be registered). “X” in two boxes for one question will be registered as a wrong answer.**
- The attached example shows you how the answer sheet would be filled in if A were the correct answer for question 1, B correct for question 2, C correct for question 3 and D correct for question 4. An “X” under E indicates that you choose not to answer question 5.
- **Your answers are to be recorded on the answer sheet. Answers written on the examination papers and not on the answer sheets will not be graded.**
- There is only one right answer for each question. Because the questions are weighted equally, it can be to your advantage to answer the simplest questions first.
- Wrong answers are given -1 point, unanswered questions get 0 points (indicated by an “X” next to E”) and correct answers are given 3 points.
- You can keep the examination papers.

**Good luck!**

## This exam has 8 questions

### QUESTION 1.

Consider the linear system

$$\begin{pmatrix} 3 & -9 & 12 & -9 & 0 \\ 0 & 2 & -4 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} -9 \\ -14 \\ 4 \\ 7 \end{pmatrix}$$

Which statement is true?

- (A) The linear system has a unique solution.
- (B) The linear system has one degree of freedom
- (C) The linear system has two degrees of freedom
- (D) The linear system is inconsistent.
- (E) I prefer not to answer.

### QUESTION 2.

Consider the vector  $\mathbf{w}$  and the set of vectors  $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ , where

$$\mathbf{w} = \begin{pmatrix} -4 \\ 3 \\ h \end{pmatrix}, \quad \mathbf{v}_1 = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 5 \\ -4 \\ -7 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$$

Which statement is true?

- (A)  $\mathbf{w}$  is a linear combination of the vectors in  $\mathcal{B}$  for all values of  $h$
- (B)  $\mathbf{w}$  is a linear combination of the vectors in  $\mathcal{B}$  exactly when  $h \neq 5$
- (C)  $\mathbf{w}$  is a linear combination of the vectors in  $\mathcal{B}$  exactly when  $h = 5$
- (D)  $\mathbf{w}$  is not a linear combination of the vectors in  $\mathcal{B}$  for any value of  $h$
- (E) I prefer not to answer.

### QUESTION 3.

Compute the rank of the matrix

$$A = \begin{pmatrix} 1 & 2 & -5 & 0 & -1 \\ 2 & 5 & -8 & 4 & 3 \\ -3 & -9 & 9 & -7 & -2 \\ 3 & 10 & -7 & 11 & 7 \end{pmatrix}$$

Which statement is true?

- (A)  $\text{rk } A = 1$
- (B)  $\text{rk } A = 2$
- (C)  $\text{rk } A = 3$
- (D)  $\text{rk } A = 4$
- (E) I prefer not to answer.

### QUESTION 4.

Consider the matrix

$$A = \begin{pmatrix} 7 & -2 \\ 2 & 3 \end{pmatrix}$$

Which statement is true?

- (A)  $A$  has eigenvalues  $\lambda = 7$  and  $\lambda = 3$
- (B)  $A$  has eigenvalues  $\lambda = 2$  and  $\lambda = -2$
- (C)  $A$  has a single eigenvalue  $\lambda = 5$
- (D)  $A$  has eigenvalues  $\lambda = 5$  and  $\lambda = -5$
- (E) I prefer not to answer.

QUESTION 5.

Consider the matrix

$$A = \begin{pmatrix} 3 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

**Which statement is true?**

- (A)  $A$  is diagonalizable with eigenvalues  $\lambda = 3$  and  $\lambda = -1$
- (B)  $A$  is diagonalizable with eigenvalues  $\lambda = 3$ ,  $\lambda = -3$  and  $\lambda = -1$
- (C)  $A$  is not diagonalizable with eigenvalues  $\lambda = 3$  and  $\lambda = -1$
- (D)  $A$  is not diagonalizable with eigenvalues  $\lambda = 3$ ,  $\lambda = -3$  and  $\lambda = -1$
- (E) I prefer not to answer.

QUESTION 6.

Consider the function

$$f(x_1, x_2, x_3) = x_1^2 + 6x_1x_2 + 3x_2^2 + 2x_3^2$$

**Which statement is true?**

- (A)  $f$  is not a quadratic form
- (B)  $f$  is a positive definite quadratic form
- (C)  $f$  is an indefinite quadratic form
- (D)  $f$  is a negative definite quadratic form
- (E) I prefer not to answer.

QUESTION 7.

Consider the function

$$f(x_1, x_2) = 3 - a \cdot Q(x_1, x_2)$$

defined on  $\mathbb{R}^2$ , where  $a \in \mathbb{R}$  is a number and  $Q$  is a positive definite quadratic form. **Which statement is true?**

- (A)  $f$  is convex for all values of  $a$
- (B)  $f$  is concave for all values of  $a$
- (C)  $f$  is convex if  $a \geq 0$  and concave if  $a \leq 0$
- (D)  $f$  is convex if  $a \leq 0$  and concave if  $a \geq 0$
- (E) I prefer not to answer.

QUESTION 8.

Consider a linear system  $A\mathbf{x} = \mathbf{0}$ , where  $A$  is a  $57 \times 61$  matrix. **Which statement is true?**

- (A) The system is inconsistent
- (B) The system has a unique solution
- (C) The system is consistent, but it is not possible to decide if the system has a unique solution or infinitely many solutions
- (D) The system has infinitely many solutions
- (E) I prefer not to answer.