A business cycle model with dynamic price

BI

$$\left.\begin{aligned} d &= a - bp \\ S &= c + dp \\ p'' &= k(d-s) \end{aligned}\right\} \Rightarrow p'' = k\big((a-bp)-(c+dp)\big)$$

$$= k(a-c) - k(b+d)p$$

$$p'' + k(b+d)p = k(a-c)$$

$$p = p_h + p_p = C_1 \cdot \cos(rt) + C_2 \cdot \sin(rt) + \frac{a-c}{b+d}$$

with $r = \sqrt{k(b+d)}$ (since $k(b+d) > 0$)

$$\bar{p} = \frac{a-c}{b+d} \longrightarrow$$

$$p(0) = C_1 \cdot \cos(0) + C_2 \cdot \sin(0) + \frac{a-c}{b+d}$$

$$= C_1 \cdot 1 + C_2 \cdot 0 + \bar{p}$$

$$p(0) = C_1 + \bar{p}$$

$$\Rightarrow \cancel{C_1 = \bar{p} - p_0} \qquad C_1 = p(0) - \bar{p}$$

$$p'(0) = -rC_1 \cdot \sin(rt) + rC_2 \cdot \cos(rt)$$

$$= -rC_1 \cdot 0 + rC_2 \cdot 1 = rC_2$$

$$\Rightarrow C_2 = p'(0)/r$$

Solution: $p(t) = C_1 \cdot \cos(rt) + C_2 \cdot \sin(rt) + \bar{p}$

with

$$\left\{\begin{aligned} C_1 &= \cancel{\bar{p} - p(0)} \quad p(0) - \bar{p} \\ C_2 &= p'(0)/r \\ \bar{p} &= \frac{a-c}{b+d} \end{aligned}\right.$$