Linear supply-demand model with dynamic price

$$\left.\begin{aligned} d &= a - bp \\ s &= c + dp \\ p' &= k(d-s) \end{aligned}\right\} \Rightarrow \quad p' = k\cdot\big((a-bp)-(c+dp)\big) = k\cdot(a-c) - k(b+d)\,p$$

$$p' + k(b+d)p = k(a-c) \qquad \underline{\text{lin. diff. eqn}}$$

<u>Solution:</u>

$$p = p_h + p_t = C\cdot e^{-rt} + \frac{k(a-c)}{k(b+d)}$$

$$= \underline{C\cdot e^{-rt} + \frac{a-c}{b+d}}$$

with $r = k\cdot(b+d) > 0$ char. root

<u>Express C in terms of p_0:</u>

$$p_0 = p(0) = C\cdot e^{0} + \frac{a-c}{b+d}$$

$$p_0 = C + \bar{p}$$

$$\Downarrow$$

$$C = p_0 - \bar{p}$$

<u>Equilibrium price:</u>

$$\bar{p} = \lim_{t\to\infty} C\cdot e^{-rt} + \frac{a-c}{b+d} = \underline{\underline{\frac{a-c}{b+d}}}$$

(since $-rt \to -\infty$)

<u>Solution:</u>

$$p(t) = C\cdot e^{-rt} + \frac{a-c}{b+d}$$

$$= \underline{\underline{(p_0-\bar{p})\,e^{-rt} + \bar{p}}} \quad \text{with} \quad \left\{\begin{aligned} r &= k\cdot(b+d) \\ \bar{p} &= \frac{a-c}{b+d} \end{aligned}\right.$$