Solution: Logistic growth model.

$$y' = ry(1 - y/K) \quad \text{with } r>0,\ K>0 \text{ parameters.}$$

General solution:

$$y' = ry(1-y/K) \quad \leftarrow \text{separable}$$

$$\frac{1}{y(1-y/K)}\, y' = r$$

$$\frac{K}{y(K-y)}\, y' = r \quad \leftarrow \text{simplified, seperated form}$$

$$\int \frac{K}{y(K-y)}\, dy = \int r\, dt$$

$$\frac{K}{y(K-y)} = \frac{A}{y} + \frac{B}{K-y}$$

$$K = A\cdot(K-y) + B\cdot y$$
$$= (-A+B)y + (AK)$$

$$-A+B=0 \qquad B=1$$
$$AK = K \qquad A=1$$

$$\frac{K}{y(K-y)} = \frac{1}{y} + \frac{1}{K-y}$$

$$\int \frac{1}{y} + \frac{1}{K-y}\, dy = \int r\, dt$$

$$\ln|y| - \ln|K-y| = rt + C_1$$

$$\ln\left|\frac{y}{K-y}\right| = rt + C_1$$

$$\left|\frac{y}{K-y}\right| = e^{rt+C_1}$$

$$\frac{y}{K-y} = \pm e^{rt+C_1} = \pm e^{C_1}\cdot e^{rt} = C\cdot e^{rt} \quad \leftarrow C = \pm e^{C_1}$$

$$y = (K-y)\cdot Ce^{rt} = KCe^{rt} - y\cdot Ce^{rt}$$

$$y(1+Ce^{rt}) = KCe^{rt} \Rightarrow \underline{\underline{y = K\cdot\frac{Ce^{rt}}{1+Ce^{rt}}}}$$

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General solution:

$$y(t) = K \cdot \frac{Ce^{rt}}{1+Ce^{rt}}$$

Initial condition: $y(0) = y_0$

$$y_0 = K \cdot \frac{C \cdot e^0}{1+Ce^0} = K \cdot \frac{C}{1+C}$$

$$y_0(1+C) = KC$$

$$C \cdot (y_0 - K) = -y_0$$

$$C = \frac{-y_0}{y_0 - K} = \underline{\underline{\frac{y_0}{K-y_0}}} \qquad \text{when } y_0 \neq K$$

Particular solution:

$$y(t) = K \cdot \frac{Ce^{rt}}{1+Ce^{rt}} \qquad \text{with } C = \frac{y_0}{K-y_0}$$

Equilibrium states:

$y' = ry(1 - y/K)$: $\quad F(y) = ry(1-y/K) = 0$

$\underline{y=0}$ or $1 - y/K = 0$

$\underline{y = K}$

$\Downarrow$

Equilibrium states: $\underline{y=0}$ and $\underline{y=K}$