FORELESNING 14

MET 1180

EIVIND ERIKSEN, NOV 18, 2015

MATEMATIKK

BI

Plan:

Repetisjon før Eksamen MET11802 (flervalgseksamen)
Gjennomgang av prøve-eksamen.

Prøve-eksamen I:

Finansmatematikk / rekker: Oppg 1–4

Geometriske rekker

$$S_n = a_1 \cdot \frac{1-k^n}{1-k}$$

(endelig, n ledd, kvot. k)

$$S = a_1 \cdot \frac{1}{1-k}$$

(uendelig, $-1 < k < 1$) dvs $|k| < 1$

1. Månedlig rente: $\frac{3\%}{12} = 0{,}25\%$

$$100.000 \cdot 1{,}0025^x = 250.000$$

$$1{,}0025^x = \frac{250.000}{100.000} = 2{,}5$$

$$\ln(1{,}0025^x) = \ln 2{,}5$$

$$x \cdot \ln 1{,}0025 = \ln 2{,}5$$

$$x = \frac{\ln 2{,}5}{\ln 1{,}0025} \simeq 366{,}97$$

$\downarrow$

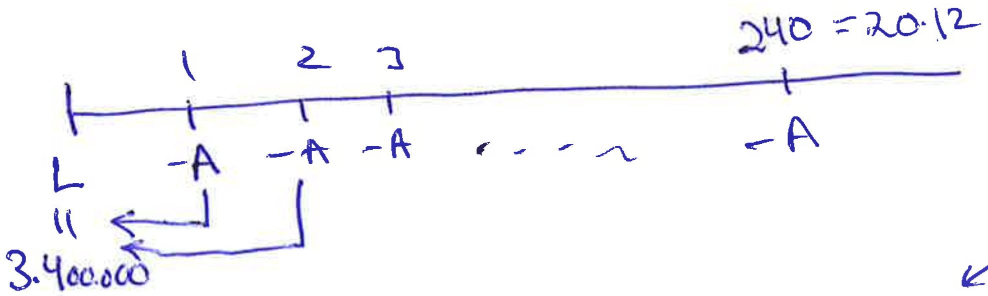
367 mnd.

$2.5\ \text{LN} \div 1{,}0025\ \text{LN} =$

30 år 7 mnd

Okt. 2035 (B)

2.



endelig geom. rekke
$n = 240$
$r = \frac{2{,}40\%}{12} = 0{,}20\%$

$$3.400.000 = \frac{A}{1+r} + \frac{A}{(1+r)^2} + \frac{A}{(1+r)^3} + \dots + \frac{A}{(1+r)^{240}}$$

$$3.400.000 = \underbrace{\frac{A}{1+r}}_{a_1} \cdot \frac{1-\left(\frac{1}{1+r}\right)^n}{1-\frac{1}{1+r}} = \frac{A \cdot \left(1-\left(\frac{1}{1+r}\right)^n\right)}{\cancel{1}+r-\cancel{1}}$$

$$3.400.000 = \frac{A \cdot \left(1-(1+r)^{-n}\right)}{r}$$

$r = 0{,}002$
$n = 240$

$$\frac{3.400.000 \cdot r}{1-(1+r)^{-n}} = A \simeq 17.851{,}52$$

$3400000\ \boxed{\times}\ 0{,}002 =$
$\div\ (1 - 1{,}002\ \boxed{y^x}\ 240$
$\pm\ ) =$

Samlede rentekostnader

$= 240 \cdot A - 3.400.000 \simeq \underline{884.365,11}$ D

svar fra forrige utregn.

$\boxed{X}\ 240 - 3400000$

3.

$S(x) = \underbrace{x}_{a_1} \underbrace{- \frac{x^3}{2}}_{a_2} + \frac{x^5}{4} - \dots$

$k = \frac{a_2}{a_1}$

$= \frac{-(x^3/2)}{x} = \underline{-\frac{x^2}{2}}$

$\underline{x = 1/2}$: $k = -\frac{(1/2)^2}{2} = \frac{-1/4}{2} = -1/8$ dvs $-1<k<1$ konvergent for $\underline{x=1/2}$

$\underset{S(1/2)}{\overset{\shortparallel}{S}} = a_1 \cdot \frac{1}{1-k} = \frac{x}{1-(-1/8)} = \frac{1/2 \cdot 8}{1+1/8 \cdot 8}$

$= \underline{\underline{\frac{4}{9}}}$

$k = -\frac{x^2}{2} = 1$: $x^2 = -2$ umulig

$k = -\frac{x^2}{2} = -1$: $x^2 = 2$ $x = \pm\sqrt{2}$

$a = \sqrt{2}$ $\boxed{C}$

$-1 < k < 1$

$-1 < -\frac{x^2}{2} < 1 \quad | \cdot (-2)$

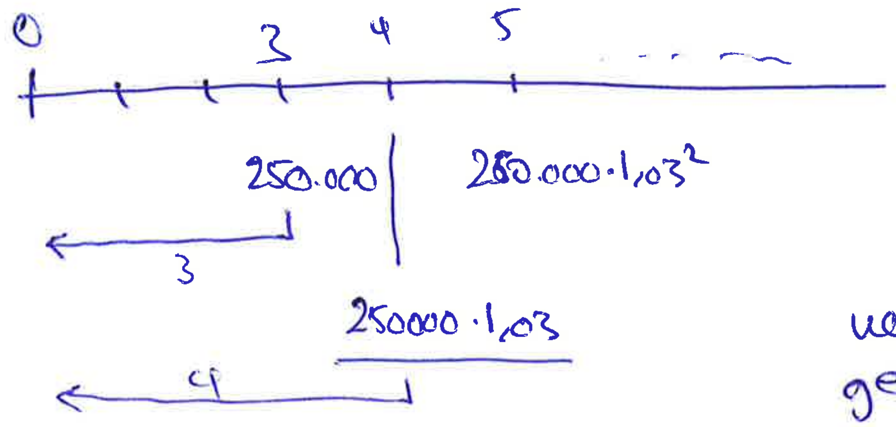
$2 > x^2 > -2$

alltid ok.

$x^2 < 2$

$-\sqrt{2} < x < \sqrt{2}$

4.



uendelig geometrisk rekke

$k = \frac{1{,}03}{1{,}08} < 1$

Nåverdi:

$$\frac{250.000}{1{,}08^3} + \frac{250.000\cdot 1{,}03}{1{,}08^4} + \frac{250.000\cdot 1{,}03^2}{1{,}08^5} + \dots$$

$$= a_1 \cdot \frac{1}{1-k} = \frac{250.000}{1{,}08^3} \cdot \frac{1}{1-\frac{1{,}03}{1{,}08}}$$

$$= \frac{250.000}{1{,}08^3 - 1{,}08^2\cdot 1{,}03} = \frac{250.000}{1{,}08^2\,(1{,}08-1{,}03)}$$

$$= \frac{250.000}{1{,}08^2\cdot 0{,}05} = 4.286.694{,}10$$

B

250.000 ÷ 0,05 = ÷ 1,08 $\boxed{y^x}$ 2 =

$$1{,}08^3\cdot\left(1-\frac{1{,}03}{1{,}08}\right) = 1{,}08^3 - \frac{1{,}03\cdot 1{,}08^{\cancel{3}\,2}}{\cancel{1{,}08}}$$

$$= 1{,}08^3 - 1{,}03\cdot 1{,}08^2$$

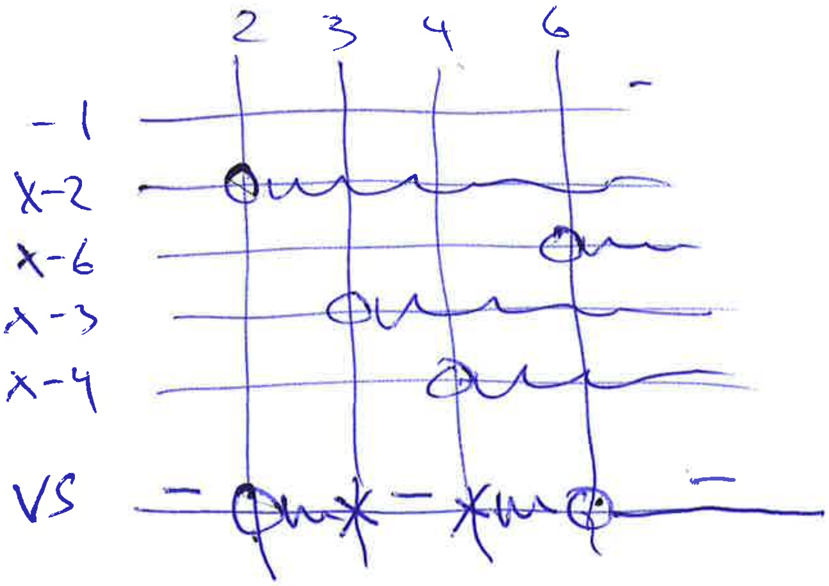
5.

$$\frac{x}{x^2-7x+12} > 1$$

$$\frac{x}{x^2-7x+12} - 1 > 0$$

$$\frac{x - (x^2-7x+12)}{x^2-7x+12} > 0$$

$$\frac{-x^2+8x-12}{x^2-7x+12} = \frac{-1\cdot(x-2)(x-6)}{(x-3)(x-4)} > 0$$



$L = (2,3) \cup (4,6)$

B

6. $x^5 - \cancel{x}\, 2x^3 = x$

$x^5 - 2x^3 - x = 0$

$x(x^4 - 2x^2 - 1) = 0$

$\underline{x=0}$ eller $x^4 - 2x^2 - 1 = 0$ ← annengradslikning i $\underline{u = x^2}$

én løsn.

$u^2 - 2u - 1 = 0$

$u = x^2 = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm \sqrt{8}}{2}$

$x^2 = \frac{2+\sqrt{8}}{2}$ eller $\cancel{x^2 = \frac{2-\sqrt{8}}{2}}$

$\underline{x = \pm\sqrt{\frac{2+\sqrt{8}}{2}}} = \pm\sqrt{1+\sqrt{2}}$ D

to løsn.

$\frac{\sqrt{8}}{2} = \sqrt{2}$

$\parallel$

$\frac{\sqrt{8}}{\sqrt{4}} = \sqrt{\frac{8}{4}} = \sqrt{2}$

7. $4 - \sqrt{x} = \sqrt{x-2}$ | $(\cdot)^2$

$(4-\sqrt{x})^2 = (\sqrt{x-2})^2$

$16 - 8\sqrt{x} + \cancel{x} = \cancel{x} - 2$

$\frac{18}{8} = \frac{\cancel{8}\sqrt{x}}{\cancel{8}}$

$\sqrt{x} = \frac{18}{8} = \frac{9}{4}$ | $(\cdot)^2$

$x = \left(\frac{9}{4}\right)^2 = \underline{\underline{\frac{81}{16}}} > 4$ B

Setter inn

HS: $\sqrt{x-2} = \sqrt{\frac{81}{16} - \frac{32}{16}} = \sqrt{\frac{49}{16}} = \underline{7/4}$

VS: $4 - \sqrt{x} = 4 - \sqrt{\frac{81}{16}} = 4 - 9/4 = \underline{7/4}$ dv.

8. $f(x) = x^3 - ax^2 - x + 3$

BI

Nullpkt i $x=a$: $f(a) = 0$

$$\cancel{a^3} - \cancel{a \cdot a^2} - a + 3 = 0$$

$$a = 3$$

$f(x) = x^3 - 3x^2 - x + 3$ har nullpkt i $x=3$

$$x^3 - 3x^2 - x + 3 : x - 3 = x^2 - 1$$
$$-(x^3 - 3x^2)$$
$$-x + 3$$
$$-(-x + 3)$$
$$0$$

$\Rightarrow f(x) = x^3 - 3x^2 - x + 3$
$= (x-3) \cdot (x^2 - 1)$

Nullpkt: $f(x) = 0$

$(x-3) \cdot (x^2-1) = 0$

$x = 3$ eller $x^2 - 1 = 0$

$x = \pm 1$

D

9.

$$f(x) = \frac{x^3 + 4x - 2}{2x^2 + x - 1} + 1$$

$$x^3 + 4x - 2 : 2x^2 + x - 1 = \frac{1}{2}x - \frac{1}{4}$$

$$-\left(x^3 + \frac{1}{2}x^2 - \frac{1}{2}x\right)$$

$$-\frac{1}{2}x^2 + \frac{9}{2}x - 2$$

$$-\left(-\frac{1}{2}x^2 - \frac{1}{4}x + \frac{1}{4}\right)$$

$$\frac{19}{4}x - \frac{9}{4}$$

Asymptoter:

Vertikale: $x = a$

hvis $x=a$ gir null i nevner, men ikke i teller

Horisontale / skrå:

polynomdivisjon

$$f(x) = \frac{x^3 + 4x - 2}{2x^2 + x - 1} + 1 = \frac{1}{2}x - \frac{1}{4} + \frac{19/4\,x - 9/4}{2x^2 + x - 1} + 1$$

$\to 0$ når $x \to \pm\infty$

$$= \frac{1}{2}x + \frac{3}{4} + \frac{\text{Rest}}{2x^2 + x - 1}$$

$y = \frac{1}{2}x + \underbrace{\frac{3}{4}}_{=b}$ er skrå asymptote

B

10. $f(x) = \frac{x\ln(x^2+1)}{(x+1)^2} = \frac{u}{v}$ $f'(0) = ?$

$$f'(x) = \frac{u'v - uv'}{v^2}$$

$$= \frac{\overbrace{\left(1\cdot\ln(x^2+1) + x\cdot\frac{1}{x^2+1}\cdot 2x\right)}^{u'}\cdot(x+1)^2 - x\ln(x^2+1)\cdot\overbrace{2(x+1)\cdot 1}^{v'}}{(x+1)^4}$$

$$f'(0) = \frac{0-0}{1^4} = 0$$

B

11. $f(x) = (x^2+x-5)e^{-x} = u\cdot v$

$f'(x) = u'v + uv'$

$= (2x+1)\cdot e^{-x} + (x^2+x-5)\cdot e^{-x}\cdot(-1)$

$= e^{-x}(2x+1-x^2-x+5)$

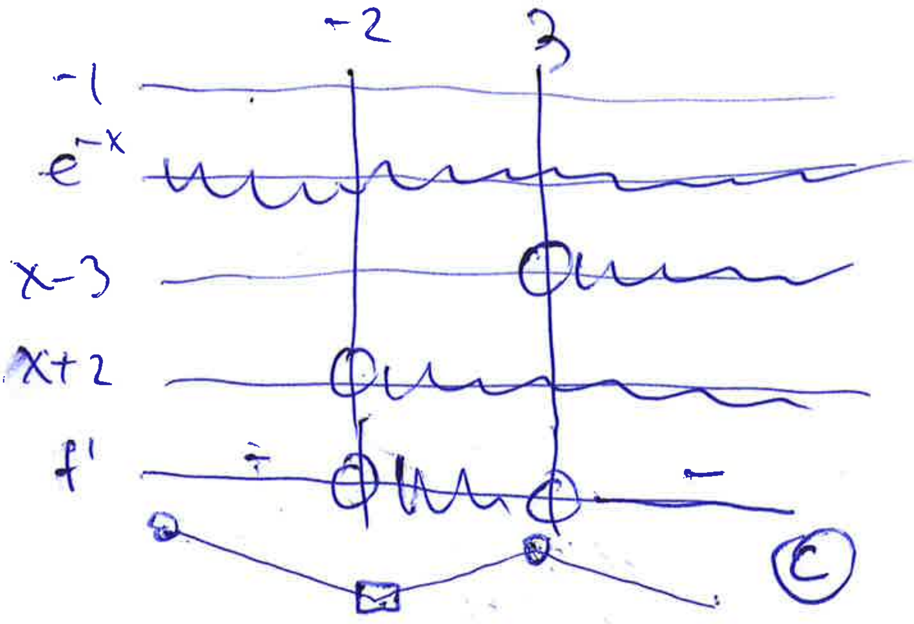
$= e^{-x}(-x^2+x+6) = -1\cdot e^{-x}(x^2-x-6)$

$= -1\cdot e^{-x}(x-3)(x+2)$

$x^2-x-6=0$

$x = \frac{1\pm\sqrt{1+24}}{2}$

$= \frac{1\pm 5}{2}$



$f(3) = 7e^{-3}$

$\lim_{x\to-\infty}(x^2+x-5)e^{-x}$

$\overset{"}{=} \infty\cdot\infty"$

$= \infty$

ingen maks

$f(-2) = -3e^{2}$

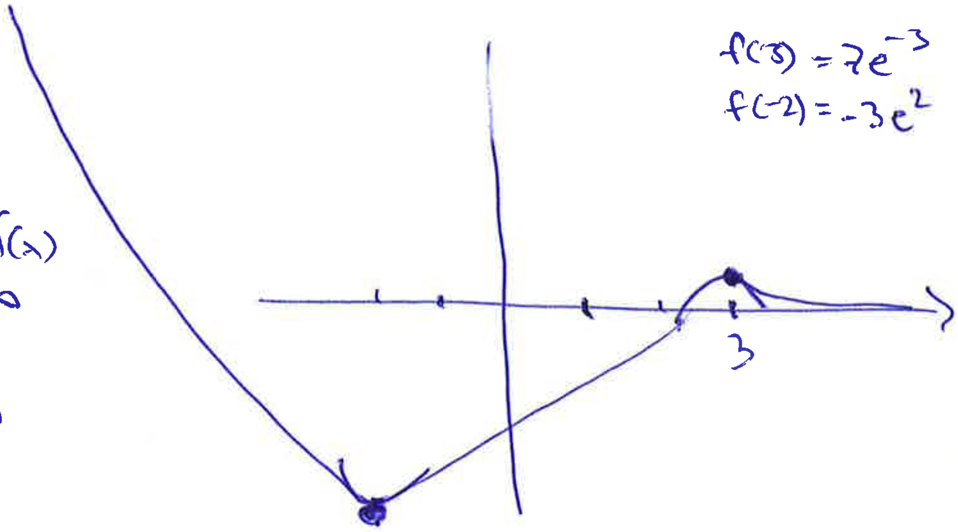
$\lim_{x\to\infty}(x^2+x-5)e^{-x}$

$= \lim_{x\to\infty}\frac{x^2+x-5}{e^x}$

$= 0$

min i $x=-2$

C



$f(3) = 7e^{-3}$
$f(-2) = -3e^{2}$

$\lim_{x \to -\infty} f(x) = \infty$

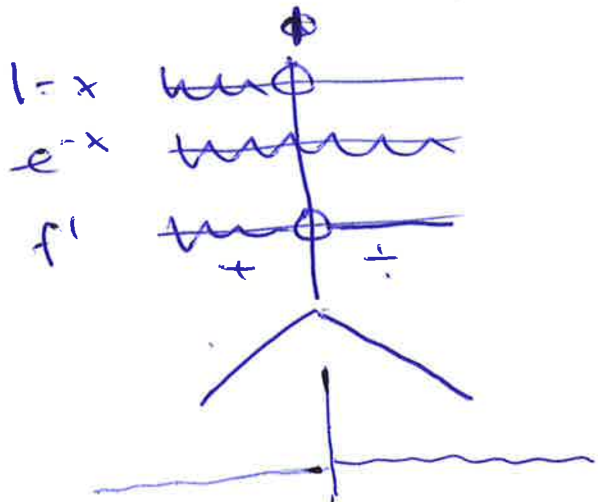
$\lim_{x \to \infty} f(x) = 0$

ingen globale maks
$x = -2$ globalt min.

12. $f(x) = x e^{-x}$, $D_f = [a, b]$

$f'(x) = 1 \cdot e^{-x} + x \cdot e^{-x} \cdot (-1)$

$= (1 - x) e^{-x}$



$x \leq 1$ voksende

$x \geq 1$ avtagende

$[-1 \ldots 1] \leftarrow$ B

$[0 \ldots 2]$

1) Hvis f er voksende på D_f eller hvis f er avtagende på D_f så fins f^{-1} (omvendt funksjon)

2) $D_{f^{-1}} = V_f$ ← y-verdier
$V_{f^{-1}} = D_f$ ← x-verdier

$D_f = [-1, 1]$

$V_f = [f(-1), f(1)]$
$= [-e, e^{-1}] = D_{f^{-1}}$

13.

$$\lim_{x\to\infty} \frac{x\ln x - \sqrt{x} + 1}{x^2} \quad \begin{matrix}\to \infty \\ \to \infty\end{matrix}$$

$$= \lim_{x\to\infty} \frac{x\ln x}{x^2} - \lim_{x\to\infty} \frac{\sqrt{x}-1}{x^2} \quad \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{\ln x}{x} \quad \text{"}\frac{\infty}{\infty}\text{"}$$

$$= \lim_{x\to\infty} \frac{1/x}{1} = \frac{0}{1} = 0$$

$$\lim_{x\to\infty} \frac{\frac{1}{2\sqrt{x}}\cdot 2\sqrt{x}}{2x\cdot 2\sqrt{x}}$$

$$= \lim_{x\to\infty} \frac{1}{4x\sqrt{x}} \quad \text{"}\frac{1}{\infty}\text{"} = 0$$

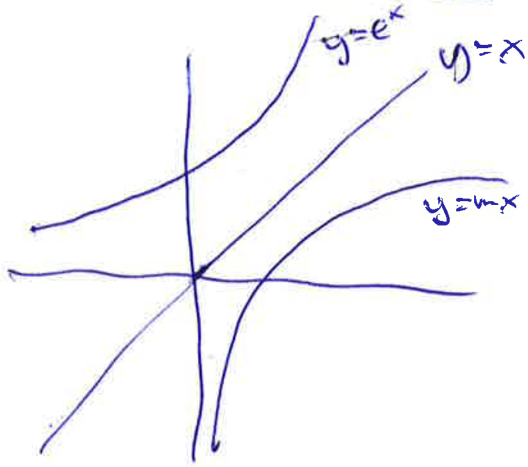
Grenseverdien $0 - 0 = 0$ (A)

L'Hôpital:

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

Hvis "0/0" eller "∞/∞"

x^2	vokser raskere enn	x
x^3	— " —	x^2
e^x	— " —	x^n
x	— " —	$\ln x$



14. $f(x) = x^2 e^x - x + 1$

$$f'(x) = \underbrace{2x\cdot e^x + x^2\cdot e^x}_{} - 1 = (2x+x^2)e^x - 1$$

$$f''(x) = \underbrace{(2+2x)e^x + (2x+x^2)\cdot e^x}_{} = (x^2+4x+2)e^x$$

$$= (x-x_1)(x-x_2)e^x$$

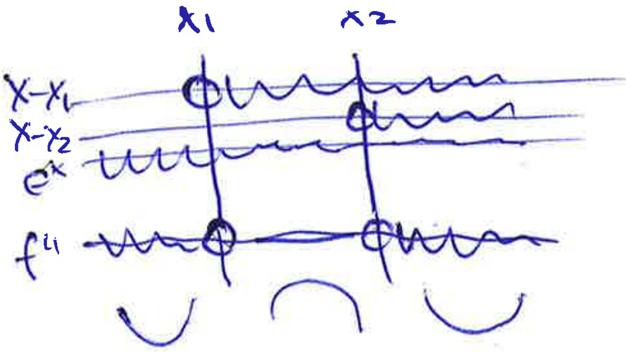
$x^2+4x+2=0$

$$x = \frac{-4\pm\sqrt{16-8}}{2}$$

$$= -2 \pm \frac{\sqrt{8}}{2} = -2\pm\sqrt{2}$$

$x_1 = -2-\sqrt{2}$

$x_2 = -2+\sqrt{2}$



To vendepkt.

(D)

15. $f(x) = \sqrt{x^3 - x}$

Lokale maks/min:

① Stasjonært: $f'(x) = 0$

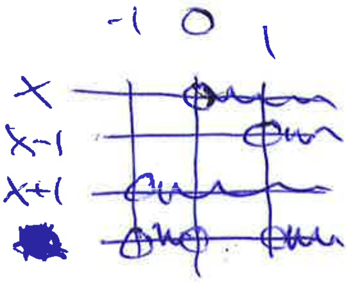
② Punkt der $f'(x)$ ikke fins.

③ Randpkt

D_f: $x^3 - x \geq 0$

$x \cdot (x^2 - 1) \geq 0$

$x(x-1)(x+1) \geq 0$



$D_f = [-1, 0] \cup [1, \infty)$

$\boxed{\text{Randpkt: } x = 0, 1, -1}$

$$f'(x) = \frac{1}{2\sqrt{x^3 - x}} \cdot (3x^2 - 1) = \frac{3x^2 - 1}{2\sqrt{x^3 - x}} = 0$$

$\boxed{\text{Punkt der } f'(x) \text{ ikke fins: } x = 0, 1, -1}$

$3x^2 - 1 = 0$

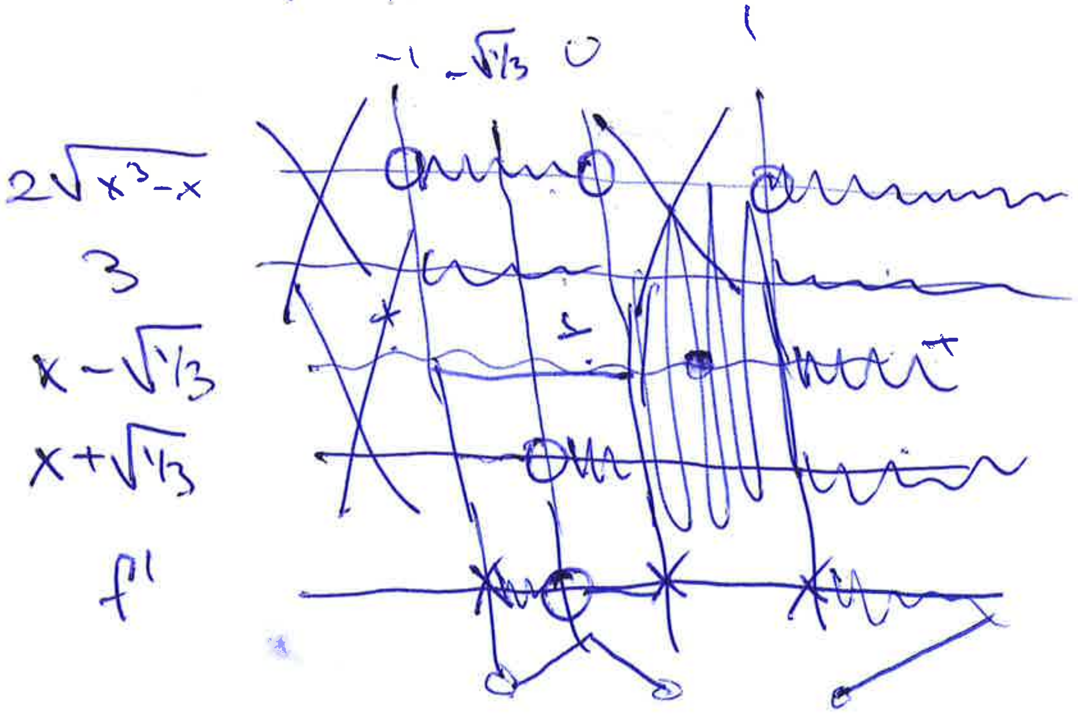
$x^2 = 1/3$

$x = \pm\sqrt{1/3} = \cancel{\pm\sqrt{1/3}}$

$= -\sqrt{1/3}$ ← Stasjonære pkt.

$\sqrt{1/3}$ ikke i D_f

$$f' = \frac{3x^2 - 1}{2\sqrt{x^3 - x}} = \frac{3(x - \sqrt{1/3})(x + \sqrt{1/3})}{2\sqrt{x^3 - x}}$$



$x = -1, 0, 1$

lokale min

$x = -\sqrt{1/3}$

lokalt maks.

(D)