PLENUMSREGNING 2

MET1180 MATEMATIKK

Eivind Eriksen, Nov 05, 2015

Plan: Eksamen 25/04/14 (MET11802)
— II — 08/05/14 (MET11803) Oppg. 4
Oppgaver fra [E]
Eksamen 11/14 og 04/15 (MET11802)

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Eksamen 25/04/14 (MET11802)

(2) $(x^2-4x-5)(x^2+6x+5) \geq 0$

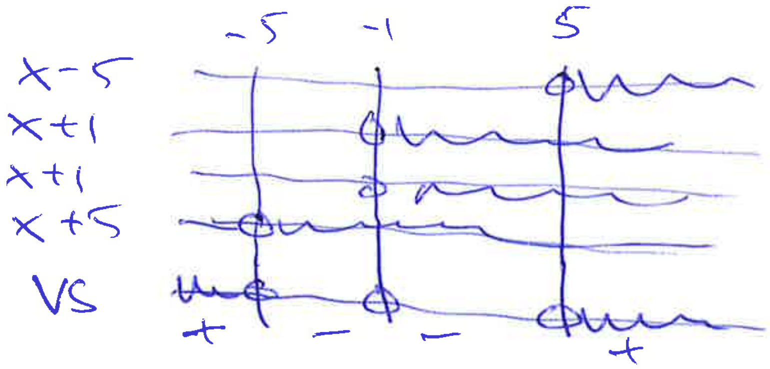
$(x-5)(x+1)(x+1)(x+5) \geq 0$

$x^2-4x-5=0$

$x = \dfrac{4 \pm \sqrt{16+20}}{2} = \dfrac{4 \pm 6}{2} = 5, -1$

$x^2+6x+5=0$

$x = \dfrac{-6 \pm \sqrt{36-20}}{2} = \dfrac{-6 \pm 4}{2} = -1, -5$



$x \leq -5$ og $x \geq 5$ og $x=-1$

$L = (-\infty, -5] \cup [5, \infty) \cup \{-1\}$ (A)

(3) $X(p) = 4p^4 + 3p^3$

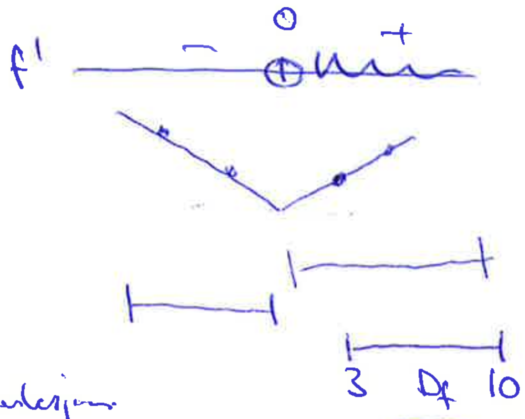
$El_p\, X(p) = \dfrac{p}{X(p)} \cdot X'(p) = \dfrac{p}{4p^4+3p^3} \cdot (16p^3+9p^2)$

$= \dfrac{16p^4+9p^3}{4p^4+3p^3} = \dfrac{16p+9}{4p+3} \underset{p=3}{=} \dfrac{57}{15} = \dfrac{19}{5}$ (C)

(5) $f(x) = e^{x^2}$

$f'(x) = e^{x^2} \cdot 2x$

↑
pos



Hvis f er strengt voksende eller strengt avtagende på Df, så har omvendt funksjon

D

(7) $(x^3 - 2ax^2 + x) : (x - a)$

$\boxed{f(x) : (x-a) \text{ har rest } = f(a)}$

Rest $= a^3 - 2a \cdot a^2 + a = -a^3 + a$

$= a \cdot (1 - a^2)$ A

Alt: $(x^3 - 2ax^2 + x) : (x - a) = x^2 - ax + (1 - a^2)$

$-(x^3 - ax^2)$

$-ax^2 + x$

$-(-ax^2 + a^2x)$

$x - a^2x$

$\shortparallel$

$(1 - a^2)x$

$-((1 - a^2)x - a(1 - a^2))$

$a(1 - a^2) = \underline{\text{Rest}}.$

(9) $e^{x^2-4x-5} = 1$

$\ln(e^{x^2-4x-5}) = \ln(1) = 0$

$x^2-4x-5=0$

$x = \frac{4 \pm \sqrt{16+20}}{2} = 2 \pm 3 = \overset{a}{\overset{\parallel}{5}}, -1$

$\sqrt{x^2+2x+1} = 25$ 625

$x^2+2x+1 = 25^2$

$x^2+2x+1-25^2 = 0$

$x^2+2x-624=0$

$x = \frac{-2 \pm \sqrt{4+2496}}{2}$

$= \frac{-2 \pm 50}{2} = -26, \overset{b}{\overset{\parallel}{24}}$

$\frac{b+1}{a} = \frac{-25}{5} = -5$ (D)

(12) Grenseinntekt = P
Grensekostnad $K'(x) = e^{3x+5} \cdot 3$

$P = 3e^{3x+5}$

$\frac{P}{3} = e^{3x+5}$

$\ln(P/3) = 3x+5$

$\frac{\ln(P/3)-5}{3} = \frac{3x}{3}$

$x = \frac{\ln(P/3)-5}{3}$ (B)

$= \frac{1}{3}\ln(P/3) - \frac{5}{3}$

(13)

$$M = \lim_{x\to 0} \frac{x+e^x \to 1}{x^2+e^{2x} \to 1} = \frac{1}{1} = \underline{1}$$

$$N = \lim_{x\to\infty} \frac{x+e^x}{x^2+e^{2x}} = \lim_{x\to\infty} \frac{1+e^x}{2x+e^{2x}\cdot 2}$$

"∞/∞"

$$= \lim_{x\to\infty} \frac{e^x}{2+e^{2x}\cdot 4} = \lim_{x\to\infty} \frac{e^x}{e^{2x}\cdot 8}$$

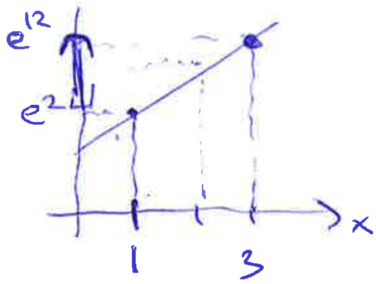
$$= \lim_{x\to\infty} \frac{\cancel{e^x}\ 1}{8\cdot e^x\cdot \cancel{e^x}} = \underline{\underline{0}}$$

$$M+N = 1+0 = 1$$ (B)

(14)

$f(x) = e^{x^2+x}$, $D_f = [1,3\rangle$

$f'(x) = e^{x^2+x}\cdot(2x+1)$



$f(1) = e^2$

$f(3) = e^{12}$ $\longrightarrow$ $V_f = [e^2, e^{12})$

(C) $\quad \| \quad D_{f^{-1}}$

Teori: $D_{f^{-1}} = V_f$

Eksamen 05/2014 MET11803

BI

(4) $3x^2 + 9x^4 + 27x^6 + \ldots$ geometrisk

$k = \frac{9x^4}{3x^2} = 3x^2$

a) $S(n) = a_1 \cdot \frac{1-k^n}{1-k}$

$= 3x^2 \cdot \frac{1-(3x^2)^n}{1-3x^2} = \frac{3x^2}{1-3x^2} \cdot (1-(3x^2)^n)$

$x = 1/3$: $S(n) = \frac{1}{3} \cdot \frac{1-(1/3)^n}{1-1/3} = \frac{1}{\cancel{3}} \cdot \frac{1-(1/3)^n}{2/\cancel{3}}$

$a_1 = 3x^2 = 3\left(\frac{1}{3}\right)^2 = 1/3$

$k = 3x^2 = 1/3$

$= \frac{1-(1/3)^n}{2}$

b) Uendelig rekke. $a_1 = 3x^2$ $k = 3x^2$

Konvergent når $-1 < k < 1$

$-1 < 3x^2 < 1$

alltid oppfyllt

$3x^2 < 1$

$x^2 < 1/3$

$-\sqrt{1/3} < x < \sqrt{1/3}$

Finn x slik at summen er 10?

$a_1 \cdot \frac{1}{1-k} = \frac{a_1}{1-k} = \boxed{\frac{3x^2}{1-3x^2} = 10} \quad \cdot (1-3x^2)$

$3x^2 = 10(1-3x^2) = 10 - 30x^2$

$33x^2 = 10 \quad x^2 = 10/33 \quad x = \pm\sqrt{\frac{10}{33}}$ Ja

$\frac{10}{33} < \frac{1}{3}$

[E] 1.6.7

0	1	2	10
-1000000	80000	$80000 \cdot 1{,}03$	$80 \cdot 1{,}03^9$

Nåverdi: $-1000000 + \dfrac{80000}{1{,}10} + \dfrac{80000 \cdot 1{,}03}{1{,}10^2} + \dots$

$$\underbrace{\dots + \frac{80.000 \cdot 1{,}03^9}{1{,}10^{10}}}_{\text{geometrisk rekke}}$$

geometrisk rekke

$n = 10$, $a_1 = \dfrac{80.000}{1{,}10}$

$k = \dfrac{1{,}03}{1{,}10}$

$$= -1000000 + a_1 \cdot \frac{1-k^n}{1-k}$$

$$= -1000.000 + \underbrace{\frac{80000}{1{,}10} \cdot \frac{1-(1{,}03/1{,}10)^{10}}{1 - 1{,}03/1{,}10}}_{\frac{80.000 \cdot (1-(1{,}03/1{,}10)^{10})}{\underbrace{(1{,}10-1{,}03)}_{=0{,}07}}}$$

kalk.

$$\simeq -449.300$$

Eksamen 11/2014 MET 11802

BI

(2)

$$3x^2 - x - 4 < x^2 - 1$$

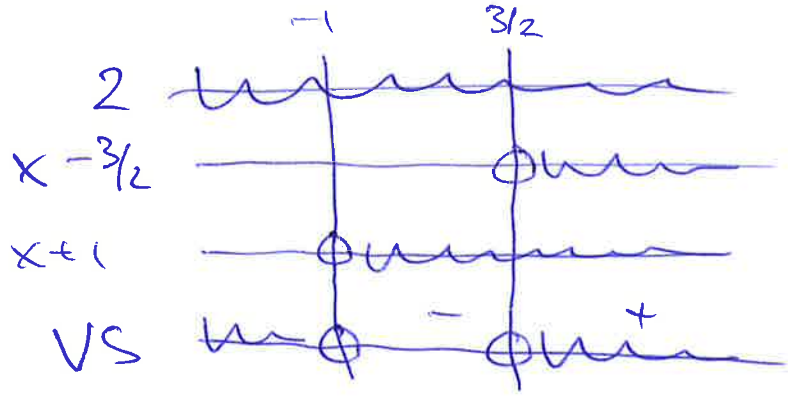
$$2x^2 - x - 3 < 0$$

$$2\left(x - 3/2\right)(x+1) < 0$$

$$2x^2 - x - 3 = 0$$

$$x = \frac{1 \pm \sqrt{1+24}}{4}$$

$$= \frac{1 \pm 5}{4} = \frac{3}{2}, -1$$



$$L = \underline{\underline{(-1, 3/2)}}$$ (B)

(4)

$$\frac{ax+1}{2} = \frac{a}{ax-1} \quad \Big| \cdot 2(ax-1)$$

$$\frac{(ax+1)\cancel{2}(ax-1)}{\cancel{2}} = \frac{a \cdot 2 \cdot \cancel{(ax-1)}}{\cancel{ax-1}}$$

$$(ax+1)(ax-1) = 2a$$

$$a^2x^2 \cancel{+ax} \cancel{-ax} - 1 = 2a$$

$$a^2x^2 = 2a+1$$

$$x^2 = \frac{2a+1}{a^2}$$

$$x = \pm\sqrt{\frac{2a+1}{a^2}}$$

(B)

(k=1) $a = -1/2$
$x = 0$
eneste løsn.

(a=0) $\frac{1}{2} = \frac{0}{-1}$
ingen løsn.
(k=0)

(a=-1) (k=0)

(5) $x^{5/4} - 1 = 1$

$$x^{5/4} = 2$$

$$(x^{5/4})^{4/5} = 2^{4/5}$$

$$a = 2^{4/5}$$

$$x = \underline{2^{4/5}} = \sqrt[5]{16}$$

$$\ln(e^{y}) = 5$$

$$y = 5 \qquad b = 5$$

$$\ln(a^{b}) = \ln\left(\left(2^{4/5}\right)^{5}\right) = \ln(2^{4}) = 4\ln(2)$$

B

(6) $$f(x) = \frac{1}{ax \cdot \ln(2ax)} = \frac{u}{v}$$

$$f'(x) = \frac{u' \cdot v - u \cdot v'}{v^2} = \frac{-\left(a \cdot \ln(2ax) + \cancel{ax} \cdot \frac{1 \cdot 2a}{\cancel{2ax}}\right)}{v^2}$$

$$= 0$$

$$-\left(a \cdot \ln(2ax) + a\right) = 0 \qquad \frac{1}{-a}$$

$$\ln(2ax) + 1 = 0$$

$$\ln(2ax) = -1$$

$$2ax = e^{-1}$$

$$x = \frac{e^{-1}}{2a} = \frac{1}{2ae} \qquad \text{A}$$

BI

9 $X(p) = 50 - p\ln(1+p)$

$$El_p X(p) = \frac{p}{X(p)} \cdot X'(p)$$

$$X'(p) = -\left(1 \cdot \ln(1+p) + p \cdot \frac{1}{1+p} \cdot 1\right)$$

$$= -\ln(1+p) - \frac{p}{1+p}$$

$$El_p X(p) = \frac{-p\left(\ln(1+p) + p/1+p\right)}{50 - p\ln(1+p)}$$

$p=6$ $$= \frac{-6\left(\ln 7 + 6/7\right)}{50 - 6\ln(7)} = \frac{-6\ln 7 - 36/7}{50 - 6\ln 7}$$

$$= -\frac{6\ln 7 + 36/7}{50 - 6\ln 7}$$

$$\simeq -\frac{17}{38}$$ mellom 0 og -1

D

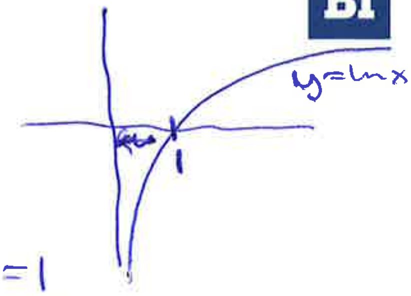
10 $(3x^4 - 2x^2 + 2) : (x-1) = 3x^3 + 3x^2 + x + 1$

$$\begin{array}{l} 3x^4 - 2x^2 + 2 \\ -(3x^4 - 3x^3) \\ \hline 3x^3 - 2x^2 + 2 \\ -(3x^3 - 3x^2) \\ \hline x^2 + 2 \\ -(x^2 - x) \\ \hline x + 2 \\ -(x - 1) \\ \hline 3 \end{array}$$

B

11

$$\lim_{x\to 0^+} \frac{\overset{\to 0}{x}}{\underset{\to 0}{\ln(x+1)}} \quad \text{"}\frac{0}{0}\text{"}$$



$$= \lim_{x\to 0^+} \frac{\overset{\to 1}{1}}{\frac{1}{x+1}\cdot 1 \to 1} = \frac{1}{1} = \underline{\underline{1}}$$

D

13

$$\sqrt{3x+2} = x+1$$

$$3x+2 = (x+1)^2 = x^2+2x+1$$

$$0 = x^2 - x - 1$$

$$x = \frac{1 \pm \sqrt{1+4}}{2}$$

$$x_1 = \frac{1+\sqrt{5}}{2} \qquad x_2 = \frac{1-\sqrt{5}}{2}$$

$$x_1 \cdot x_2 = \left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right)\left(\frac{1}{2} - \frac{\sqrt{5}}{2}\right) = \frac{1}{4} - \frac{5}{4} = \frac{-4}{4}$$

B $\quad = \underline{\underline{-1}}$

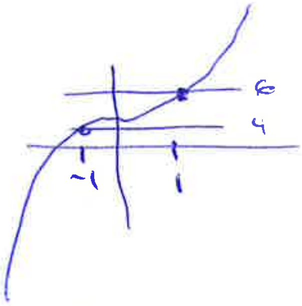
Eksamen 04/2015 MET 11802

BI

(1) $f = x^3 + 5$, $D_f = [-1, 1]$

$D_{f^{-1}} = V_f = [4, 6]$

B



$f' = 3x^2 \geq 0$

$f(-1) = 4$ $f(1) = 6$

(8) $K(x) = \ln(x^2+1)$

$K'(x) = \frac{1}{x^2+1} \cdot 2x = \frac{2x}{x^2+1}$

Når er $\frac{2x}{x^2+1}$ maksimal?

$h(x) = \frac{2x}{x^2+1}$. Finn maks.

$h'(x) = \frac{2 \cdot (x^2+1) - 2x \cdot 2x}{(x^2+1)^2} = \frac{2x^2+2-4x^2}{(1+x^2)^2}$

$= \frac{2-2x^2}{(1+x^2)^2} = 0$

$2 - 2x^2 = 0$

$x^2 = 1$

$x = 1$

C

(14) $g = e^{3x+2}$

$g' = e^{3x+2} \cdot 3$

$$\frac{g'}{g} = \frac{3 \cdot \cancel{e^{3x+2}}}{\cancel{e^{3x+2}}} = \underline{3}$$ (C)

(15) $I'(x) = (x(20-x))'$

$= (20x - x^2)' = 20 - 2x$

$K'(x) = (x^2+6x+1)' = 2x + 6$

$2x + 6 = 20 - 2x$

$\frac{4x}{4} = \frac{14}{4}$

$x = 14/4 = \underline{7/2}$ (A)

Eks: $\cancel{x^3} + 7x^2 - x + 3 : \underline{3x^2} - x + 4 = \frac{x}{3} + \frac{22}{9}$

$\frac{x^3}{3x^2} =$

$-\left(\cancel{x^3} - \frac{x^2}{3} + \frac{4x}{3}\right)$

$\cancel{\frac{22}{3}x^2} - \frac{7}{3}x + 3$

$-\left(\cancel{\frac{22}{3}x^2} - \frac{22}{9}x + \frac{88}{9}\right)$

$\frac{1}{9}x - \frac{61}{9} \leftarrow$ Rest

$7 + \frac{1}{3}$

$\frac{21}{3} + \frac{1}{3}$

$\frac{22}{9} - \frac{21}{9}$

$27/9 - 88/9$

$y = \frac{1}{3}x + \frac{22}{9}$

er skrå asymptote