	Plan
1	Repetisjon og oppgavegjennomgang
2	Integrasjon av rasjonale uttrykk

(1) Oppgaver

MET11805 12/2022:

6. $D(p) = (p+20)e^{-0.05p}, \quad p > 0$

$$\varepsilon = \frac{p}{D(p)} \cdot D'(p) = \frac{p}{(p+20)e^{-0.05p}} \cdot \left(1 \cdot e^{-0.05p} + (p+20) \cdot e^{-0.05p} \cdot (-0.05)\right)$$

$$= \frac{p \cdot \cancel{e^{-0.05p}} \cdot (\cancel{1} + p \cdot (-0.05) \cancel{+ 20 \cdot (-0.05)})}{(p+20) \cdot \cancel{e^{-0.05p}}}$$

$$= \frac{p^2 \cdot (-0.05)}{p+20} = \underline{\underline{\frac{-0.05p^2}{p+20}}}$$

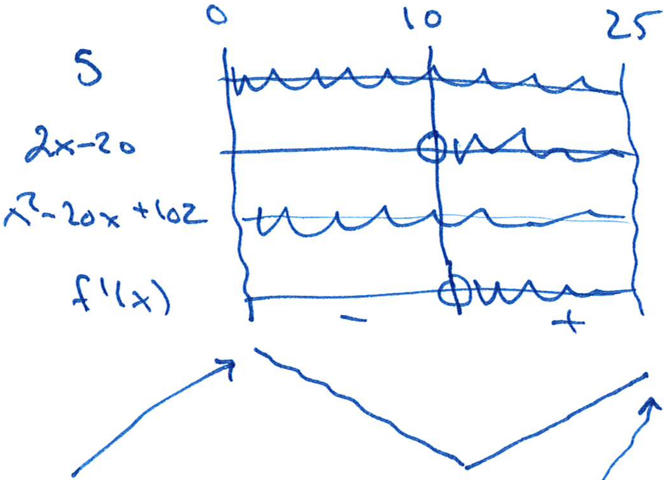
$p = 40$: $\varepsilon(40) = \dfrac{-0.05 \cdot 40^2}{60} = -\dfrac{80}{60} \approx \underline{\underline{-1.33}}$

$5\ln(u)$, $u = x^2 - 20x + 102$

9. $f(x) = 5\ln(x^2 - 20x + 102)$, $0 \le x \le 25$

$$f'(x) = 5 \cdot \frac{1}{u} \cdot u' = \frac{5(2x-20)}{x^2-20x+102} = 0 \qquad 2x - 20 = 0$$

$$\underline{x = 10}$$



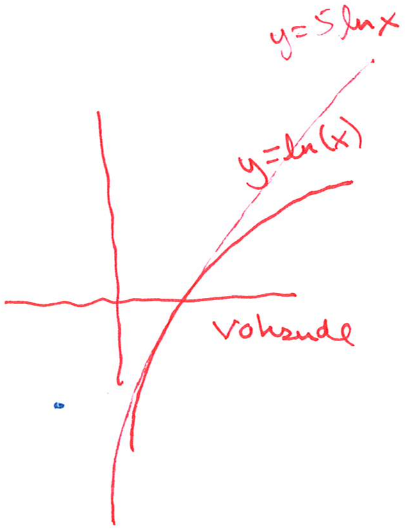
$x^2 - 20x + 102$
$= (x-10)^2 + 2 > 0$

Min: $\underline{x = 10}$ $f_{min} = f(10) = \underline{\underline{5\ln 2}}$

Max: $\underline{x = 25}$ $f_{max} = \underline{\underline{5\ln(227)}}$

$\underline{x=0}$: $f(0) = 5\ln(102)$

$\underline{x=25}$: $f(25) = 5\ln(227)$



Alt: $f(x) = 5\ln(x^2-20x+102)$, $0 \le x \le 25$

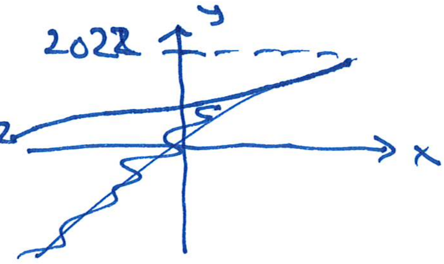
$$= 5\ln\left(\underbrace{(x-10)^2+2}\right)$$

finn største
og minste verdi
av det kvadratiske
uttrykket

11. $f(x) = \dfrac{2022 e^x}{e^x+1}$, $D_f = \mathbb{R} = (-\infty, \infty)$

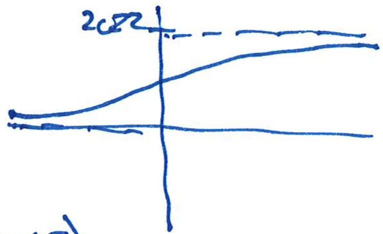
$$f'(x) = 2022 \cdot \left(\frac{e^x(\cancel{e^x}+1) - \cancel{e^x \cdot e^x}}{(e^x+1)^2} \right) = \frac{2022 \cdot e^x}{(e^x+1)^2} > 0$$

$V_f = \underline{(0, 2022)}$



$$\lim_{x \to \infty} \frac{2022 e^x}{e^x+1} = 2022 \cdot \lim_{x\to\infty} \frac{e^x}{e^x+1} \overset{L'H.}{=} 2022 \cdot \lim_{x \to \infty} \frac{e^x}{e^x} = 2022$$

$$\lim_{x \to -\infty} \frac{2022 e^x}{e^x+1} = \frac{2022 \cdot 0}{0+1} = \frac{0}{1} = \underline{0}$$



$D_g = V_f = \underline{\underline{(0, 2022)}}$ $\quad V_g = D_f = \underline{\underline{(-\infty, \infty)}}$

Uttrykket for $g(x)$:

$$f(x) = \frac{2022 e^x}{e^x+1}$$

$$y = \frac{2022 e^x}{e^x+1} \quad \Big| \cdot (e^x+1)$$

$$y \cdot (e^x+1) = 2022 e^x$$

$$y \cdot e^x - 2022 e^x = -y$$

$$\frac{(y-2022)e^x}{y-2022} = \frac{-y}{y-2022}$$

$$e^x = \frac{-y}{y-2022} \cdot \frac{(-1)}{(-1)} = \frac{y}{2022-y}$$

$$x = \ln\left(\frac{y}{2022-y} \right)$$

$\rightarrow g(x) = \ln\left(\dfrac{x}{2022-x} \right)$, $\quad 0 < x < 2022$

Oppgaveark 26

9h

$$\int \frac{\ln x}{x}\,dx = \int \frac{u}{\cancel{x}} \cdot \cancel{x}\,du$$

$$\boxed{\begin{aligned} u &= \ln x \\ du &= \tfrac{1}{x}\,dx \end{aligned}} \rightarrow dx = x \cdot du$$

$$= \int u\,du = \tfrac{1}{2}u^2 + C = \underline{\underline{\tfrac{1}{2}(\ln x)^2 + C}}$$

Alt:

$$\int \frac{\ln x}{x}\,dx = \int \frac{1}{x} \cdot \ln x\,dx$$

$$\boxed{\begin{array}{ll} u = \ln x & v = \ln x \\ u' = 1/x & v' = 1/x \end{array}}$$

$$= (\ln x)^2 - \int \frac{1}{x} \cdot \ln x\,dx$$

$I = \int \frac{1}{x} \cdot \ln x\,dx$:

$$I = (\ln x)^2 - I$$

$$2I = (\ln x)^2$$

$$I = \frac{(\ln x)^2}{2} + C$$

$$= \underline{\underline{\tfrac{1}{2}(\ln x)^2 + C}}$$

12. Anta $f(x) \geq 0$ for alle x og at $\int f(x)\,dx = F(x) + C$

Er $F(x)$ en voksende funksjon?

$F'(x) = f(x) \geq 0 \implies F(x)$ er <u>voksende fn.</u>

Repetisjon:

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C \quad (n \neq -1)$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

Delvis:

$$\int u'v\,dx = uv - \int uv'\,dx$$

Substitusjon:

$$du = u'\,dx$$

② Integrasjon av rasjonale uttrykk

Rasjonalt uttrykk: $\dfrac{p(x)}{q(x)}$ der $p(x)$, $q(x)$ er polynomer

Eks: $\dfrac{x}{x-1}$, $\dfrac{x^2}{x+3}$, $\dfrac{x^2-4}{x^2-3x+2}$

Enkleste tilfelle: $\boxed{\text{Nevneren har grad 1.}}$

~~Derivasjonsregel:~~ $\cancel{\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}}$

Eks:

i) $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$

ii) $\displaystyle\int \frac{1}{x-2}\,dx = \ln|x-2| + C$

iii) $\displaystyle\int \frac{1}{1-x}\,dx \neq \ln|1-x| + C$

iv) $\displaystyle\int \frac{2}{4-2x}\,dx \neq 2\cdot\ln|4-2x| + C$

Bruk substitusjon: $u = \text{nevneren}$

ii) $\displaystyle\int \frac{1}{x-2}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \underline{\underline{\ln|x-2| + C}}$

$\boxed{\begin{array}{l} u = x-2 \\ du = 1\cdot dx \end{array}}$

iii) $\displaystyle\int \frac{1}{1-x}\,dx = \int \frac{1}{u}\left(\frac{1}{-1}\right)du = \frac{1}{-1}\int \frac{1}{u}\,du = \underline{\underline{-\ln|1-x| + C}}$

$\boxed{\begin{array}{l} u = 1-x \\ du = -1\cdot dx \end{array}} \rightarrow dx = \frac{1}{-1}\cdot du$

iv) $\int \frac{2}{4-2x}\,dx = \int \frac{2}{u}\left(\frac{1}{-2}\right)du = \frac{2}{-2}\int \frac{1}{u}\,du$

$\boxed{\begin{aligned} u &= 4-2x \\ du &= -2\cdot dx \end{aligned}} \to dx = \frac{1}{-2}\,du$

$= -\ln|u| + C = -\ln|4-2x| + C$

Formel:

$$\boxed{\int \frac{A}{ax+b}\,dx = \frac{A}{a}\cdot \ln|ax+b| + C \quad (a \neq 0)}$$

$\boxed{\begin{aligned} u &= ax+b \\ du &= a\,dx \end{aligned}} \to dx = \frac{1}{a}\,du$

$\to = \int \frac{A}{u}\left(\frac{1}{a}\right)du = \frac{A}{a}\int \frac{1}{u}\,du$

$= \frac{A}{a}\ln|u| + C$

$= \frac{A}{a}\ln|ax+b| + C$

Eks: $\int \frac{x}{x-1}\,dx$

$= \int 1 + \frac{1}{x-1}\,dx$

$= x + \frac{1}{1}\ln|x-1| + C$

$= x + \ln|x-1| + C$

$\int \frac{x^3}{x-1}\,dx$

$= \int x^2 + x + 1 + \frac{1}{x-1}\,dx$

$= \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + \ln|x-1| + C$

Polynomdivisjon:

$x : x-1 = 1$
$x-1$
1

$\frac{x}{x-1} = 1 + \frac{1}{x-1}$

$x^3 : x-1 = x^2+x+1$
$x^3 - x^2$
x^2
$x^2 - x$
x
$x-1$
1

$\frac{x^3}{x-1} = x^2+x+1 + \frac{1}{x-1}$