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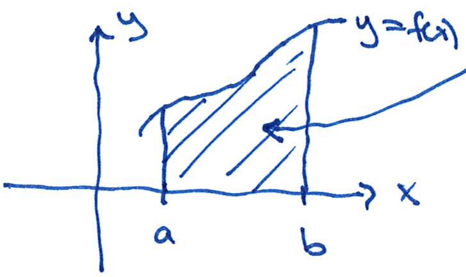
① Bestemte integral som areal

Teorem:

Hvis $f(x)$ er kont. i $[a,b]$ og $f(x) \geq 0$ i $[a,b]$ så har vi

$$\int_a^b f(x)\,dx = \left\{ \begin{array}{l} \text{arealet under grafen} \\ \text{til } f(x) \text{ i } [a,b] \end{array} \right\}$$



$$\int_1^2 \ln x\,dx = \Big[x \cdot \ln x - x\Big]_1^2$$
$$= (2\ln 2 - 2) - (\cancel{1 \cdot \ln 1} - 1)$$
$$= 2\ln 2 - 1 \simeq \underline{0.386}$$

Riemannsum:

Riemannsum $n = 4$, sluttpkt.

$$0.25 \cdot \ln(1.25) + 0.25 \cdot \ln(1.5) + 0.25 \cdot \ln(1.75) + 0.25 \ln(2)$$

Merk:

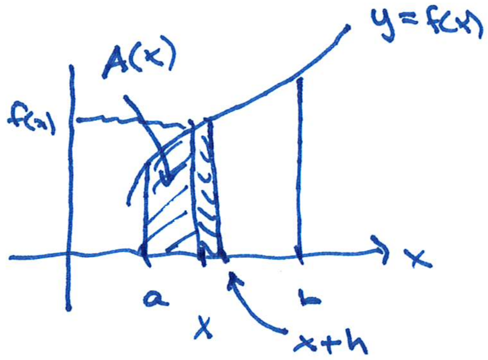
Når $n \to \infty$ vil Riemannsummen nærme seg det virkelige arealet.

n = antall delintervall = 4

$x_0 = 1 \quad x_1 = 1.25 \quad x_2 = 1.5 \quad x_3 = 1.75 \quad x_4 = 2$

Forklaring av teoremet:

$$\left\{ \begin{matrix} \text{arealet under } f \\ \text{på } [a,b] \end{matrix} \right\} = \int_a^b f(x)\,dx$$



For $a \le x \le b$, definerer vi

$A(x) =$ arealet under grafen til f i $[a,x]$

Vet: 1) $A(a) = 0$
2) $A(b) = A$ ← arealet vi vil finne
3) $A'(x) = f(x)$

$$A'(x) \approx \frac{\Delta A(x)}{\Delta x} = \frac{A(x+h)-A(x)}{h}$$

$$= \frac{\text{arealet mellom } x \text{ og } x+h}{h}$$

$$\approx \frac{\cancel{h}\cdot f(x)}{\cancel{h}} = f(x)$$

Det betyr:

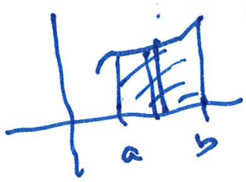
$$\int_a^b f(x)\,dx = \Big[A(x)\Big]_a^b$$

$$= A(b) - A(a) = A - 0 = \underline{\underline{A}}$$

$$\boxed{\int_a^b f(x)\,dx = \text{summen av } f(x)\cdot\Delta x \text{ når } a \le x \le b}$$

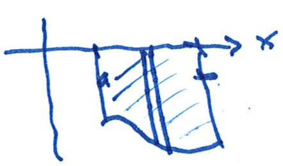
Arealberegning: $f(x)$, $g(x)$ kont. på $[a,b]$

i) $f(x) \ge 0$ i $[a,b]$



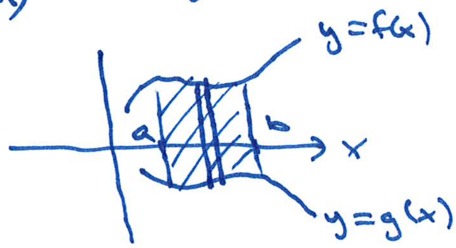
$$A = \int_a^b f(x)\,dx$$

ii) $f(x) \le 0$ i $[a,b]$



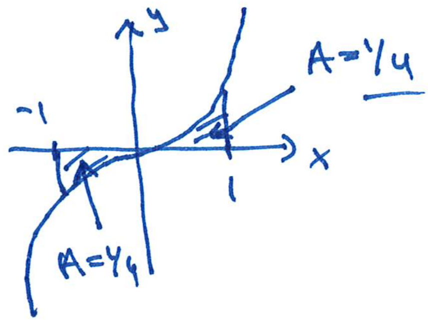
$$A = -\int_a^b f(x)\,dx = \int_a^b -f(x)\,dx$$

iii) $f(x) \ge g(x)$ i $[a,b]$



$$A = \int_a^b f(x) - g(x)\cdot dx$$

Eks: $\int_{-1}^{1} x^3\,dx = \left[\frac{1}{4}x^4\right]_{-1}^{1} = 0$



$$\int_{-1}^{0} x^3\,dx + \int_{0}^{1} x^3\,dx = \left[\frac{1}{4}x^4\right]_{-1}^{0} + \left[\frac{1}{4}x^4\right]_{0}^{1}$$

$$= \underbrace{\left(\cancel{\tfrac{1}{4}\cdot 0^4}\right) - \left(\tfrac{1}{4}\cdot(-1)^4\right)} + \underbrace{\left(\tfrac{1}{4}\cdot 1^4 - \cancel{\tfrac{1}{4}\cdot 0^4}\right)}$$

$$= -1/4 + 1/4 = 0$$

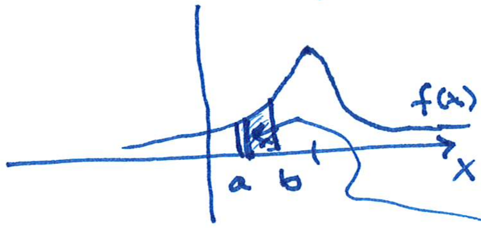
Areal: $-\int_{-1}^{0} x^3\,dx + \int_{0}^{1} x^3\,dx = -(-1/4) + (1/4)$

$$= \underline{\underline{1/2}}$$

$$\boxed{\begin{array}{l} \underline{\text{Generelt:}} \\ \text{i)}\ \int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx \\ \text{ii)}\ \int_a^b f(x)\,dx = -\int_b^a f(x)\,dx \end{array}}$$

2 Økonomiske anvendelser av integrasjon

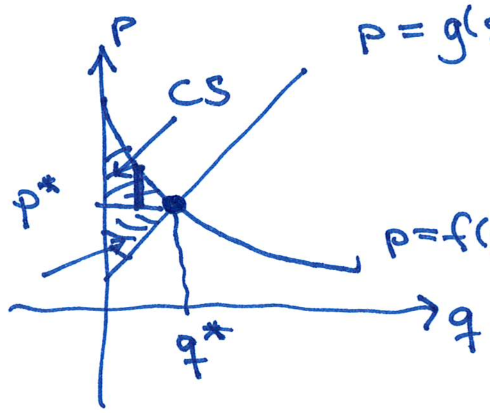
i) Kontinuerlige stokastiske variabler



$f(x)$ = tetthetsfunksjonen til normal-fordelingen

$$P(a \le x \le b) = \int_a^b f(x)\,dx = \text{arealet}$$

ii) Konsument/produsentoverskudd



$p = g(q)$ (omvendt) tilbudsfunksjon

$p = f(q)$ (omvendt) etterspørselsfn.

$$CS = \text{konsumentoverskudd} = \int_0^{q^*} f(q) - p^* \, dq$$

$$PS = \text{produsentoverskudd} = \int_0^{q^*} p^* - g(q) \, dq$$

3) Kontant strømmer

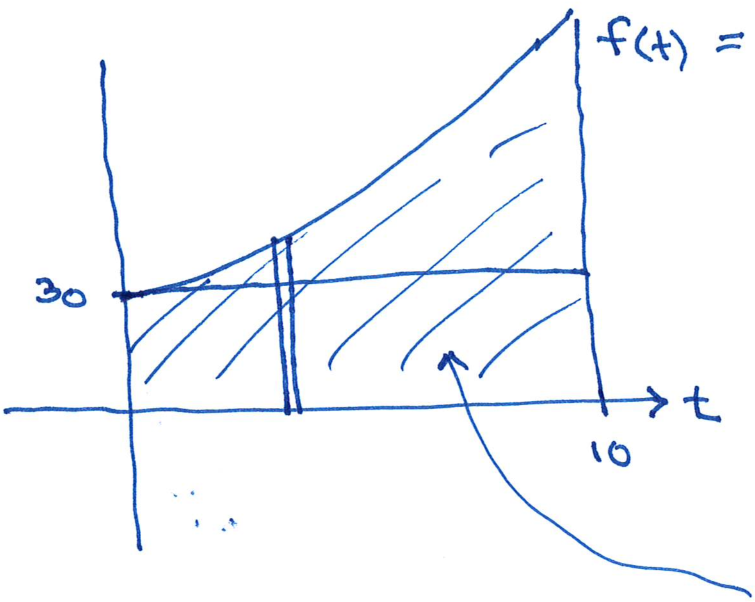
Eks: Du har en leieinntekt på 30 MNOK/år, og at leieinntekten øker med 6% per år

Samlet leieinntekt i 10 år:

$f(t) = 30 \cdot e^{0.06t}$

t = tiden (år)
Ser på $[0,10]$

$f(t)$ = renteinntekt per år



$$\int_0^{10} 30 \cdot e^{0.06t}\, dt = \text{areal} = \text{samlet leieinntekt i 10 år}$$

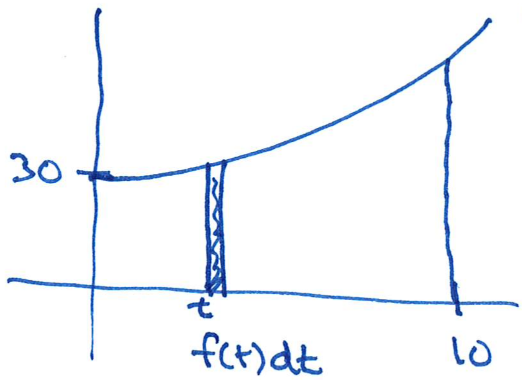
$$\boxed{\begin{aligned} u &= 0.06t \\ du &= 0.06 \cdot dt \end{aligned}}$$

$$= \int_0^{0.6} 30 \cdot e^u \cdot \frac{1}{0.06}\, du = \frac{30}{0.06}\left[e^u\right]_0^{0.6} = 500\left(e^{0.6} - e^0\right)$$

$$= 500\left(e^{0.6} - 1\right) \simeq \underline{411.1} \text{ MNOK}$$

Nåverdi: Kontinuerlig diskontering, $r = 10\%$

$f(t) = 30 \cdot e^{0.06t}$



$f(t)dt$

$$[f(t)\,dt]\, e^{-0.10t} = \frac{30\, e^{0.06t}\, dt}{e^{0.10t}} = \underbrace{30 \cdot e^{0.06t} \cdot e^{-0.10t}}\, dt$$

Nåverdi (samlet):

$$\int_0^{10} 30\, \underbrace{e^{0.06t} \cdot e^{-0.10t}}\, dt = \int_0^{10} f(t) \cdot e^{-0.10t}\, dt$$

$$= \int_0^{10} 30 \cdot e^{-0.04t}\, dt = \int_0^{-0.4} 30 \cdot e^u\, du \cdot \frac{1}{-0.04}$$

$$\boxed{\begin{array}{l} u = -0.04t \\ du = -0.04\, dt \end{array}}$$

$$= \frac{30}{-0.04} \Big[e^u \Big]_0^{-0.4} = -750 \left(e^{-0.4} - e^0 \right)$$

$$= 750 \left(1 - \frac{1}{e^{0.4}} \right) \cong \underline{\underline{247.3}} \text{ MNOK}$$

Formler:

Samlet inntekt:	$\int_{t_1}^{t_2} f(t)\, dt$	$f(t)$: inntekt/år
Samlet nåverdi:	$\int_{t_1}^{t_2} f(t) \cdot e^{-rt}\, dt$	r: diskonteringsrente
		$[t_1, t_2]$: tidsintervall

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