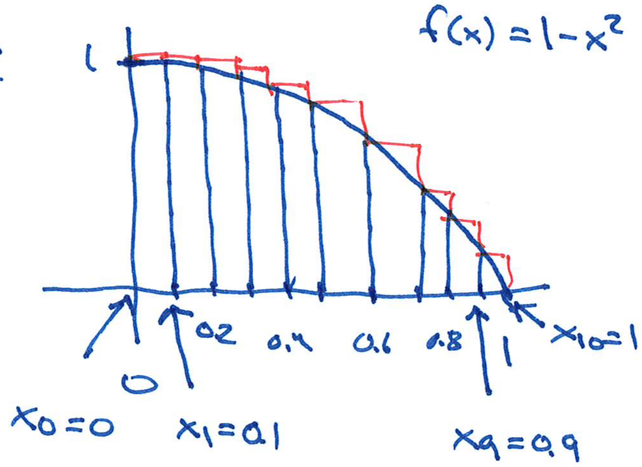
Plan

1 Systemer av likninger
2 Lineære likningssystemer

Oppgaveark 30

9. $f(x) = 1 - x^2$



$$A = \int_0^1 1 - x^2\,dx$$

$$= \left[x - \tfrac{1}{3}x^3\right]_0^1 = \underline{\underline{2/3}}$$

$n = 10 \qquad \Delta x = \frac{1-0}{10} = 0.1$

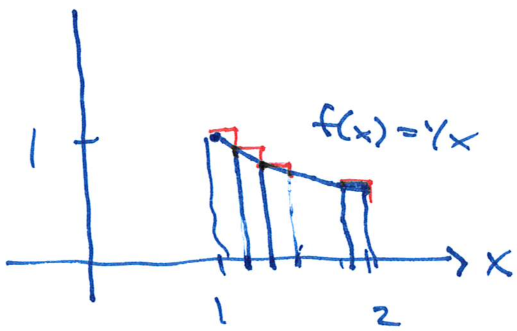
Startpkt.

$$R = 0.1 \cdot f(0) + 0.1 \cdot f(0.1) + \dots + 0.1 \cdot f(0.9)$$

$$= 0.1 \cdot \left[1 + (1 - 0.1^2) + (1 - 0.2^2) + \dots + (1 - 0.9^2)\right]$$

$$= 0.715$$

10. $f(x) = 1/x$



$$A = \int_1^2 1/x\,dx = \left[\ln|x|\right]_1^2$$

$$= \ln 2 - \ln 1 = \underline{\underline{\ln 2}} \approx 0.69$$

n delintervall $\quad \Delta x = \frac{2-1}{n} = 1/n$

Startpkt

$$R_n = \frac{1}{n} \cdot f(1) + \frac{1}{n} \cdot f(1 + 1/n) + \frac{1}{n} \cdot f(1 + 2/n) + \dots + \frac{1}{n} \cdot f\left(1 + \frac{n-1}{n}\right)$$

$$= \frac{1}{n} \cdot \frac{1}{1} + \frac{1}{n} \cdot \frac{1}{1 + 1/n} + \dots + \frac{1}{n} \cdot \frac{1}{1 + \frac{n-1}{n}}$$

$$= 1/n + \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n-1}$$

Vis at

$$\lim_{n \to \infty} \left(\frac{1}{n} + \frac{1}{n+1} + \dots + \frac{1}{2n-1} \right) = \ln 2$$

① Systemer av likninger

Eks: (1) $x+y=4$, $x-y=2$ — 2x2 likningssystem

(2) $x^2+y^2=10$, $x+y=4$ — 2x2 likn.sys.

Innsettingsmetoden

Eks: (1)

$$x+y=4$$
$$x-y=2$$

$y=4-x$

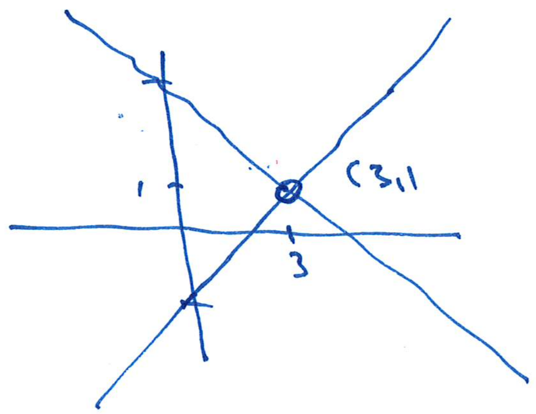
$x-(4-x)=2$

$x-4+x=2$

$2x=6$

$x=3$ $\qquad y=4-3=1$

Løsn: $(x,y)=(3,1)$ én løsn.



$x+y=4$ grad 1

$y=4-x$

$x-y=2$ grad 1

$y=x-2$

(2)

$$x^2+y^2=10$$
$$x+y=4$$

$\boxed{x^2+(4-x)^2=10}$

$y=4-x$

$x^2+16-8x+x^2=10$

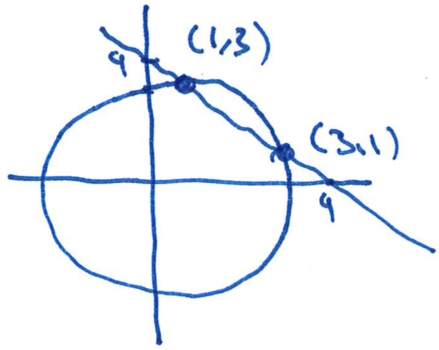
$2x^2-8x+6=0 \quad |:2$

$x^2-4x+3=0$

$x=3$ eller $x=1$

$y=1$ $\qquad$ $y=3$

Løsn: $(x,y)=(3,1),\ (1,3)$ (to løsn.)

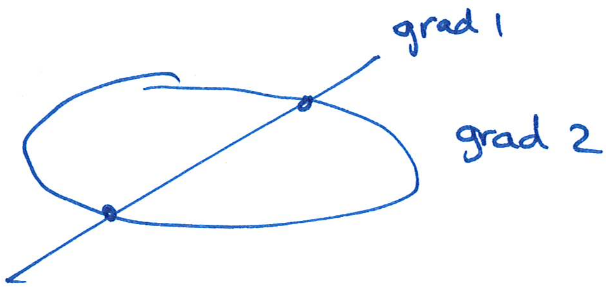


$x^2+y^2=10$ grad 2

sirkel, sentr $(0,0)$, $r=\sqrt{10}$

$x+y=4$ grad 1

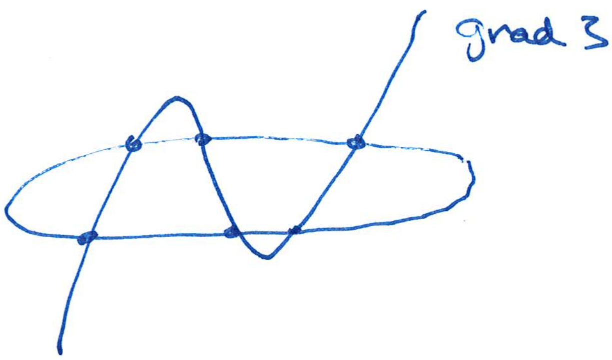
$y=4-x$ rett linje



Bezout's teorem

To likninger i to variabler som er polynomlikninger av grad d_1 og d_2 har vanligvis $d_1 \cdot d_2$ løsninger

$\left.\begin{array}{l}\text{grad 2}\\ \text{grad 3}\end{array}\right\}$ $2\cdot 3=6$ løsn.



(2) Lineære likningssystemer

Defn: Et likn. system er lineært hvis alle likn. er lineære (grad 1).

Et $m \times n$ lineært system:

m lineære likn.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \vdots \qquad\qquad\qquad\qquad \vdots \qquad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{cases}$$

n ukjente $(x_1, x_2, \dots, x_n)$

der $a_{11}, \dots, a_{mn}$, og $b_1, \dots, b_m$ er gitte tall.

Eks:

$$\begin{aligned} x+y+z &= 1 \\ x+2y+4z &= 4 \\ x+3y+9z &= 11 \end{aligned}$$

3×3 lineært system

Metode: Gauss-eliminasjon (generell metode for å løse alle lineære system)

Eks:

$$x+y+z=1$$
$$x+2y+4z=4$$
$$x+3y+9z=11$$

Eks:

$$\begin{aligned} x+y&=4 \\ x-y&=2 \\ \hline + \quad 2x &= 6 \end{aligned} \qquad \begin{aligned} x+y&=4 \\ x-y&=2 \\ \hline \div \quad 2y &= 2 \end{aligned}$$

$$\begin{array}{cccc} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 4 \\ 1 & 3 & 9 & 11 \end{array}\right) \\ x\text{-kol.} \quad y\text{-kol.} \quad z\text{-kol.} \end{array}$$

utvidet matrise

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{pmatrix}$$

koeffisient-matrisen

Elementære radoperasjoner

ønsker null her

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 4 \\ 1 & 3 & 9 & 11 \end{array}\right) \; -1$$

ny rad 2 = rad 2 + $\underbrace{(-1)\cdot \text{rad } 1}$

$(-1 \; -1 \; -1 \; | -1)$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 3 \\ 1 & 3 & 9 & 11 \end{array}\right) \; -1$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 3 \\ 0 & 2 & 8 & 10 \end{array}\right) \; -2$$

ønsker null her

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 2 & 4 \end{array}\right)$$

trappeform

Defn:

Pivot = første tall ulik null i en rad

elementære radoperasjoner:

i) bytte om to rader

ii) mult. en rad med et tall $c \neq 0$

iii) legge til et multiplum av en rad til en annen rad

trappeform:

- alle nullrader nederst
- alle pivoter står lenger til høyre enn pivotene i radene overfor

$$\begin{aligned} \underline{x} + y + z &= 1 \\ \underline{y} + 3z &= 3 \\ \underline{2z} &= 4 \end{aligned}$$

Baklengs substitusjon:

(3) $2z = 4 \quad \underline{z = 2}$

(2) $y + 3z = 3$

$y + 3(2) = 3 \quad \underline{y = -3}$

(1) $x + y + z = 1$

$x + (-3) + 2 = 1 \quad \underline{x = 2}$

Konklusjon: $(x, y, z) = \underline{\underline{(2, -3, 2)}}$ (én løsn)

Oppsummering: Gauss-eliminasjon

i) Skriv ned den utvidede matrisen til det lineære systemet

ii) Bruk elementære radoperasjoner til vi har en trappeform (alltid mulig, mer enn et svar!)

iii) Skriv trappeformen tilbake til et lineært system, og løs vha baklengs substitusjon.

Eks:

$$\left(\begin{array}{ccc|c} 0 & 4 & 7 & 3 \\ 1 & 2 & 7 & 4 \\ 2 & -1 & 0 & 7 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & 7 & 4 \\ 0 & 4 & 7 & 3 \\ 2 & -1 & 0 & 7 \end{array}\right)$$

Eks:

$$x+y+z=3$$
$$x+2y+4z=7$$
$$2x+3y+5z=10$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 2 & 4 & 7 \\ 2 & 3 & 5 & 10 \end{array}\right) \begin{matrix} \\ -1 \\ -2 \end{matrix} \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 3 & 4 \\ 0 & 1 & 3 & 4 \end{array}\right) \begin{matrix} \\ \\ -1 \end{matrix}$$

$$\to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

trappeform

$$\underline{x}+y+z=3 \qquad x = 3-(4-3z)-z$$
$$= -1+2z$$
$$\underline{y}+3z=4 \qquad y=4-3z$$

$$(x,y,z) = \underline{(-1+2z,\ 4-3z,\ z)}$$

der z er en fri variabel

uendelig mange løsn.