	Plan
1	Repetisjon og oppgavegjennomgang
2	Noen nyttige regneregler for matriser
3	Inverse matriser: Tilfellet $n = 2$

1 Repetisjon: Regning med matriser
- matrisemultiplikasjon
- transponering

2 Regneregler for matriser:

For determinant:
i) $|A \cdot B| = |A| \cdot |B|$ *
ii) $|A^T| = |A|$
iii) $|c \cdot A| = c^n \cdot |A|$ (c tall, A nxn-matr.)

For transponering:
i) $(AB)^T = B^T \cdot A^T$ *
ii) $(A^T)^T = A$
iii) $(A + c \cdot B)^T = A^T + c \cdot B^T$

Eivind Eriksen, Kontor B4-032, eivind.eriksen@bi.no
http://www.dr-eriksen.no/teaching/MET1180/

Oppgaveark 35

6. $\underline{b}$ er en lineær-komb. av $\underline{v}_1, \underline{v}_2, \underline{v}_3$ $\iff$ $x\begin{pmatrix}1\\1\\1\\1\end{pmatrix} + y\begin{pmatrix}1\\2\\4\\3\end{pmatrix} + z\cdot\begin{pmatrix}1\\-1\\1\\7\end{pmatrix} = \begin{pmatrix}a\\b\\c\\d\end{pmatrix}$

$\uparrow \underline{v}_1$ $\uparrow \underline{v}_2$ $\uparrow \underline{v}_3$ $\uparrow \underline{b}$

er konsistent (har løsn. for x, y, z)

$\Updownarrow$

ingen pivotpos. i siste kolonne i trappeformen til $(\underline{v}_1|\underline{v}_2|\underline{v}_3|\underline{b})$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & a \\ 1 & 2 & -1 & b \\ 1 & 4 & 1 & c \\ 1 & 3 & 7 & d \end{array}\right) \begin{matrix} \\ -1 \\ -1 \\ -1 \end{matrix}$$

$$\to \left(\begin{array}{ccc|c} 1 & 1 & 1 & a \\ 0 & 1 & -2 & b-a \\ 0 & 3 & 0 & c-a \\ 0 & 2 & 6 & d-a \end{array}\right) \begin{matrix} \\ \\ -3 \\ -2 \end{matrix}$$

$$\to \left(\begin{array}{ccc|c} 1 & 1 & 1 & a \\ 0 & 1 & -2 & b-a \\ 0 & 0 & 6 & c-a-3(b-a) \\ 0 & 0 & 10 & d-a-2(b-a) \end{array}\right) \begin{matrix} \\ \\ \\ -\frac{10}{6} \end{matrix}$$

$$\to \left(\begin{array}{ccc|c} 1 & 1 & 1 & a \\ 0 & 1 & -2 & b-a \\ 0 & 0 & 6 & * \\ 0 & 0 & 0 & d-a-2(b-a)-\frac{10}{6}\left(c-a-3(b-a)\right) \end{array}\right)$$

$\Updownarrow$

6.1 $d-a-2(b-a)-\frac{10}{6}(c-a)+5(b-a)=0$

$\underline{6d}-\underline{6a}-\underline{12b}+\underline{12a}-\underline{10c}+\underline{10a}+\underline{30b}-30a=0$

$-14a+18b-10c+6d=0 \quad |:(-2)$

$\underline{\underline{7a-9b+5c-3d=0}}$

$(a,b,c,d)=(0,0,1,1)$: $0-0+5\cdot 1-3\cdot 1=2\neq 0$

$\Rightarrow$ $\underline{\underline{\text{ikke en lineær komb}}}$

7.) b)

	A	B	C
1	20	5	30
2	40	-50	180
3	-20	25	-265

← gevinst per aksje

($C = 400.000$)

$$\left.\begin{aligned} 20x + 5y + 30z &= R_1 \\ 40x - 50y + 180z &= R_2 \\ -20x + 25y - 265z &= R_3 \\ 60x + 75y + 320z &= C \end{aligned}\right\}$$

løser ved hjelp av Gauss

$$\left(\begin{array}{ccc|c} 20 & 5 & 30 & R_1 \\ 40 & -50 & 180 & R_2 \\ -20 & 25 & -265 & R_3 \\ 60 & 75 & 320 & C \end{array}\right) \begin{matrix} -2 \\ 1 \\ -3 \end{matrix} \to \left(\begin{array}{ccc|c} 20 & 5 & 30 & R_1 \\ 0 & -60 & 120 & R_2 - 2R_1 \\ 0 & 30 & -235 & R_3 + R_1 \\ 0 & 60 & 230 & C - 3R_1 \end{array}\right) \begin{matrix} 1/2 \\ 1 \end{matrix}$$

$$\to \left(\begin{array}{ccc|c} 20 & 5 & 30 & R_1 \\ 0 & -60 & 120 & R_2 - 2R_1 \\ 0 & 0 & -175 & R_3 + \cancel{R_1} + \frac{1}{2}(R_2 - \cancel{2R_1}) \\ 0 & 0 & 350 & C - 3R_1 + (R_2 - 2R_1) \end{array}\right) 2$$

$$\to \left(\begin{array}{ccc|c} 20 & 5 & 30 & R_1 \\ 0 & -60 & 120 & R_2 - 2R_1 \\ 0 & 0 & -175 & R_3 + \frac{1}{2}R_2 \\ 0 & 0 & 0 & C - 5R_1 + R_2 + 2(R_3 + \frac{1}{2}R_2) \end{array}\right)$$

$$\underbrace{C - 5R_1 + R_2 + 2(R_3 + \tfrac{1}{2}R_2)}_{=\; C - 5R_1 + 2R_2 + 2R_3 ?}$$

① Mulig med avkastningene $(R_1, R_2, R_3) \iff \boxed{5R_1 - 2R_2 - 2R_3 = \overset{400.000}{\overset{\shortparallel}{C}}}$

② Finn porteføljen ved å løse for x, y, z.

b) $(R_1, R_2, R_3) = (50', 25', -100')$: $5 \cdot 50' - 2 \cdot 25' - 2 \cdot (-100') = 400'$ ok

$$\begin{aligned} 20x + 5y + 30z &= 50' \\ -60y + 120z &= -75' \\ -175z &= -87.5' \end{aligned}$$

$z = \underline{500}$

$y = \dfrac{-135'}{-60} = \underline{2250}$

$x = \underline{1187.5}$

c) $R_1 > 0,\ R_2, R_3 = 0$: $R_2 = R_3 = 0$ $5R_1 = 400'$

$R_1 = 80'$

$\Rightarrow (80', 0, 0)$ er mulig.

$$20x + 5y + 30z = 80'$$
$$-60y + 120z = -160'$$
$$-175z = 0$$

$x = 3333\ 1/3$

$y = \frac{160'}{60} = 2666\ 2/3$

$z = 0$

d) $R_1, R_2, R_3 > 0$: $5R_1 - 2R_2 - 2R_3 = 400.000$

$R_1 = R_2 = R_3$ $5R_1 - 2R_1 - 2R_1 = 400'$

$R_1 = 400'$

Ja, $(R_1, R_2, R_3) =$

$(400', 400', 400')$

$\cancel{R_1 = 60'}$

$R_1 = 100'$ $R_2 = R_3 = 25'$ Ja, $(100', 25', 25')$ går også.

9. $A = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}$ $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\frac{1}{a} \cdot |\ ax = 1$

$x = 1/a$

a) $AX = I$: $\begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} a+3c & b+3d \\ 2a+5c & 2b+5d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{aligned} a \quad +3c \quad &= 1 \\ b \quad +3d &= 0 \\ 2a \quad +5c \quad &= 0 \\ 2b \quad +5d &= 1 \end{aligned}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 1 \\ 0 & 1 & 0 & 3 & 0 \\ 2 & 0 & 5 & 0 & 0 \\ 0 & 2 & 0 & 5 & 1 \end{array}\right) \begin{matrix} \\ -2 \\ \\ -2 \end{matrix}$$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 1 \\ 0 & 1 & 0 & 3 & 0 \\ 0 & 0 & -1 & 0 & -2 \\ 0 & 0 & 0 & -1 & 1 \end{array}\right)$$

$a = -5$
$b = 3$
$c = 2$
$d = -1$

$$X = \begin{pmatrix} -5 & 3 \\ 2 & -1 \end{pmatrix} = A^{-1}$$

b) $X^2 = A \Rightarrow |X^2| = |A| = -1 \Rightarrow |X|^2 = -1$ umulig

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\cdot\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix} \quad \Big| \, |\cdot|$$

$$\left|\begin{pmatrix} a^2+bc & b(a+d) \\ c(a+d) & d^2+bc \end{pmatrix}\right| = \left|\begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}\right|$$

$$(a^2+bc)(d^2+bc) - bc(a^2+2ad+d^2) = -1$$

$$\underline{a^2d^2} + \cancel{bcd^2} + \cancel{a^2bc} + \underline{b^2c^2} - \cancel{bca^2} - \underline{2bcad} - \cancel{bcd^2} = -1$$

$$a^2d^2 - 2\,ad\cdot bc + b^2c^2 = -1$$

$$(ad - bc)^2 = -1 \quad \text{umulig} \quad \text{ingen løsning}$$

$$|X|^2 = -1$$

c) $AX = XA$

$$\begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}\cdot\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\cdot\begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}$$

$$\begin{pmatrix} a+3c & b+3d \\ 2a+5c & 2b+5d \end{pmatrix} = \begin{pmatrix} a+2b & 3a+5b \\ c+2d & 3c+5d \end{pmatrix}$$

c, d fri

$$X = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -2c+d & 3/2\, c \\ c & d \end{pmatrix} = c\cdot\begin{pmatrix} -2 & 3/2 \\ 1 & 0 \end{pmatrix} + d\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\cancel{a} + 3c = \cancel{a} + 2b \qquad -2b + 3c = 0$$

$$b + 3d = 3a + 5b \qquad -3a - 4b + 3d = 0$$

$$2a + 5c = c + 2d \qquad 2a + 4c - 2d = 0$$

$$2b + \cancel{5d} = 3c + \cancel{5d} \qquad 2b - 3c = 0$$

$2a+4c-2d=0 \quad \underline{a = -2c+d}$

$-4b + 6c = 0 \Rightarrow \underline{b = \frac{3}{2}c}$

$$\left(\begin{array}{cccc|c} 0 & -2 & 3 & 0 & 0 \\ -3 & -4 & 0 & 3 & 0 \\ 2 & 0 & 4 & -2 & 0 \\ \cancel{0} & \cancel{2} & \cancel{-3} & \cancel{0} & \cancel{0} \end{array}\right) \to \begin{pmatrix} 2 & 0 & 4 & -2 \\ -3 & -4 & 0 & 3 \\ 0 & -2 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \; 3/2 \to \left(\begin{array}{cccc|c} 2 & 0 & 4 & -2 & 0 \\ 0 & -4 & 6 & 0 & 0 \\ \cancel{0} & \cancel{-2} & \cancel{3} & \cancel{0} & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & 0 \end{array}\right)$$

c, d fri

Litt liten tid til c), her er utregningen i mer detalj:

$$\left(\begin{array}{cccc|c} 0 & -2 & 3 & 0 & 0 \\ -3 & -4 & 0 & 3 & 0 \\ 2 & 0 & 4 & -2 & 0 \\ 0 & 2 & -3 & 0 & 0 \end{array}\right) \to \left(\begin{array}{cccc|c} 2 & 0 & 4 & -2 & 0 \\ -3 & -4 & 0 & 3 & 0 \\ 0 & -2 & 3 & 0 & 0 \\ 0 & 2 & -3 & 0 & 0 \end{array}\right) 3/2$$

Utvidet matrise (se forrige side)

bytter om for å få pivot øverst til venstre

$$\to \left(\begin{array}{cccc|c} 2 & 0 & 4 & -2 & 0 \\ 0 & -4 & 6 & 0 & 0 \\ 0 & -2 & 3 & 0 & 0 \\ 0 & 2 & -3 & 0 & 0 \end{array}\right) \begin{array}{c} -1/2 \\ 1/2 \end{array} \to \left(\begin{array}{cccc|c} 2 & 0 & 4 & -2 & 0 \\ 0 & -4 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

c, d fri

$2a + 4c - 2d = 0$

$-4b + 6c = 0$

$\Downarrow$

$\frac{-4b}{-4} = \frac{-6c}{-4} \Rightarrow b = \frac{6}{4}c = \frac{3}{2}c$

$\frac{2a}{2} = \frac{-4c+2d}{2} \Rightarrow a = -2c + d$

Løsning:

$(a,b,c,d) = (-2c+d, \frac{3}{2}c, c, d)$ med c, d fri

$= (-2c, \frac{3}{2}c, c, 0) + (d, 0, 0, d)$

$= \frac{c}{2} \cdot (-4, 3, 2, 0) + d(1, 0, 0, 1)$

$X = \frac{c}{2} \cdot \begin{pmatrix} -4 & 3 \\ 2 & 0 \end{pmatrix} + d \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= s \cdot \begin{pmatrix} -4 & 3 \\ 2 & 0 \end{pmatrix} + t \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

husk: $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

(3) Inverse matriser: Ikke tid i dag, kommer torsdag.

(Kan utsette Oppgavesett 36 til torsdag).