... if I couldn't formulate a problem in economic theory mathematically, I didn't know what I was doing. I came to the position that mathematical analysis is not one of the many ways of doing economic theory: it is the only way.

R. Lucas

Forelesning 28
Kap 5.5-6: Delbrøksoppspaltning. Bestemte integraler.

Lærebokoppgaver

[L] 5.5.1-3
[L] 5.6.1-2

Oppgaver for veiledningstimene torsdag 18/1 fra 12 i D1-065/70

Oppgave 1.

Regn ut disse ubestemte integralene:

a) $\int \dfrac{4}{4-x}\, dx$
b) $\int \dfrac{4}{4-x^2}\, dx$
c) $\int \dfrac{4x}{4-x^2}\, dx$
d) $\int \dfrac{x^2}{4-x^2}\, dx$

Oppgave 2.

Regn ut disse ubestemte integralene:

a) $\int \dfrac{1}{1-x^2}\, dx$
b) $\int \dfrac{2x}{1-x^2}\, dx$
c) $\int \dfrac{x^2}{1-x^2}\, dx$
d) $\int \dfrac{x^2-2x+1}{1-x^2}\, dx$

e) $\int \dfrac{1}{(1-x)^2}\, dx$
f) $\int \dfrac{2x}{(1-x)^2}\, dx$
g) $\int \dfrac{x^2}{(1-x)^2}\, dx$
h) $\int \dfrac{x^2-2x+1}{(1-x)^2}\, dx$

Oppgave 3.

Løs de bestemte integralene:

a) $\int_0^1 x\, dx$
b) $\int_0^1 x^2\, dx$
c) $\int_0^1 x^3\, dx$
d) $\int_0^1 e^x\, dx$

e) $\int_0^1 (e^x + e^{-x})\, dx$
f) $\int_{-1}^1 x\, dx$
g) $\int_{-1}^1 x^2\, dx$
h) $\int_{-1}^1 x^3\, dx$

i) $\int_{-1}^1 e^x\, dx$
j) $\int_{-1}^1 (e^x + e^{-x})\, dx$

Oppgave 4.

Løs de bestemte integralene:

a) $\displaystyle\int_0^1 xe^x \, \mathrm{d}x$ b) $\displaystyle\int_0^1 x\ln(x^2+1) \, \mathrm{d}x$ c) $\displaystyle\int_0^1 \frac{1}{x^2+5x+6} \, \mathrm{d}x$ d) $\displaystyle\int_0^1 \frac{1}{x^2+4x+4} \, \mathrm{d}x$

e) $\displaystyle\int_{-1}^1 xe^x \, \mathrm{d}x$ f) $\displaystyle\int_{-1}^1 x\ln(x^2+1) \, \mathrm{d}x$ g) $\displaystyle\int_{-1}^1 \frac{1}{x^2+5x+6} \, \mathrm{d}x$ h) $\displaystyle\int_{-1}^1 \frac{1}{x^2+4x+4} \, \mathrm{d}x$

Oppgave 5.

Eksamen MET1180 (Desember 2015) Oppgave 2abc

Regn ut disse ubestemte integralene:

a) $\displaystyle\int xe^{1-x^2} \, \mathrm{d}x$ b) $\displaystyle\int x\ln(1-x) \, \mathrm{d}x$ c) $\displaystyle\int \frac{x^3+x^2-2x-6}{x^2-1} \, \mathrm{d}x$

Oppgave 6.

Eksamen MET1180 (Mai 2016) Oppgave 3abc

Regn ut disse ubestemte integralene:

a) $\displaystyle\int \frac{\ln x + 1}{x^2} \, \mathrm{d}x$ b) $\displaystyle\int x^3 \sqrt{x^2+4} \, \mathrm{d}x$ c) $\displaystyle\int \frac{x^2}{x^2+5x+4} \, \mathrm{d}x$

Oppgave 7.

Regn ut disse ubestemte integralene:

a) $\displaystyle\int 2x^3 e^{-x^2} \, \mathrm{d}x$ b) $\displaystyle\int \sqrt{x}\, e^{\sqrt{x}} \, \mathrm{d}x$ c) $\displaystyle\int \frac{\sqrt{x}+1}{1-\sqrt{x}} \, \mathrm{d}x$

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Oppgave 1.

    a) $-4\ln|4-x| + C$ b) $\ln|2+x| - \ln|2-x| + C$

    c) $-2\ln|2-x| - 2\ln|2+x| + C$ d) $-x + \ln|2+x| - \ln|2-x| + C$

Oppgave 2.

    a) $\dfrac{1}{2}\ln\left|\dfrac{1+x}{1-x}\right| + C$ b) $-\ln|1-x^2| + C$ c) $-x + \dfrac{1}{2}\ln\left|\dfrac{1+x}{1-x}\right| + C$

    d) $-x + 2\ln|1+x| + C$ e) $\dfrac{1}{1-x} + C$ f) $2\ln|1-x| + \dfrac{2}{1-x} + C$

    g) $x + 2\ln|1-x| + \dfrac{1}{1-x} + C$ h) $x + C$

Oppgave 3.

a) $1/2$ b) $1/3$ c) $1/4$ d) $e-1$

e) $e-1/e$ f) 0 g) $2/3$ h) 0

i) $e-1/e$ j) $2(e-1/e)$

Oppgave 4.

a) 1 b) $\ln(2)-1/2$ c) $2\ln(3)-3\ln(2)$ d) $1/6$

e) $2/e$ f) 0 g) $\ln(3)-\ln((2)$ h) $2/3$

Oppgave 5.

a) $-\frac{1}{2}e^{1-x^2}+C$ b) $\frac{1}{2}x^2\ln(1-x)-\frac{1}{2}x-\frac{1}{4}x^2-\frac{1}{2}\ln(1-x)+C$

c) $\frac{1}{2}x^2+x-3\ln|x-1|+2\ln|x+1|+\mathscr{C}$

Oppgave 6.

a) $-\frac{1}{x}(\ln x+2)+C$ b) $\frac{1}{5}(x^2+4)^{5/2}-\frac{4}{3}(x^2+4)^{3/2}+C$

c) $x-\frac{16}{3}\ln|x+4|+\frac{1}{3}\ln|x+1|+C$

Oppgave 7.

a) $-x^2e^{-x^2}-e^{-x^2}+C$ b) $2xe^{\sqrt{x}}-4\sqrt{x}e^{\sqrt{x}}+4e^{\sqrt{x}}+C$

c) $5-4\sqrt{x}-x-4\ln|1-\sqrt{x}|+C$