# Løsning Prøve-eksamen Nov 2015, Alt. 2

**1.**

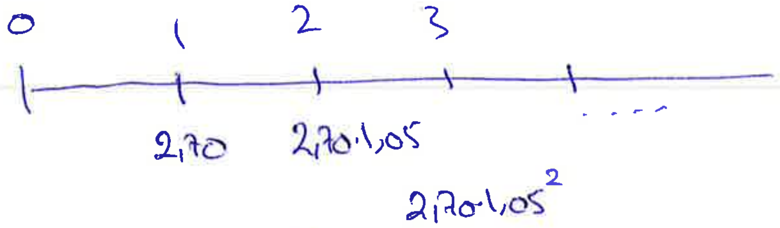
$$100.000 \cdot \left(e^{0,03}\right)^x = 145.000$$

$$e^{0,03x} = \frac{145.000}{100.000} = 1,45$$

$$0,03x = \ln(1,45)$$

$$x = \frac{\ln(1,45)}{0,03} \simeq \underline{12,40} > 12 \qquad \text{D}$$

**2.**




$$\text{Nåverdi: } \frac{2,70}{1,10} + \frac{2,70\cdot 1,05}{1,10^2} + \frac{2,70\cdot 1,05^2}{1,10^3} + \dots \qquad \text{geometrisk} \quad k = \frac{1,05}{1,10} < 1$$

$a_1$

$$= \frac{2,70}{1,10} \cdot \frac{1}{1 - 1,05/1,10}$$

$$= \frac{2,70}{1,10 - 1,05} = \frac{2,70}{0,05} = \underline{54} \qquad \text{C}$$

**3.**

$$S(x) = \frac{18}{x} - 2x^2 + \frac{2x^5}{9} - \dots$$

$$\text{Geometrisk}, \quad k = \frac{-2x^2}{18/x} = -\frac{2}{18}\frac{x^2}{x^{-1}} = \underline{-\frac{1}{9}x^3}$$

$$\underline{x=3}: \quad k = -\frac{1}{9}\cdot 3^3 = -\frac{1}{9}\cdot 27 = -3 < -1$$

Rekken er <u>divergent</u> (ikke konvergent)

A

4.

0 1 2 3 ....

Inv. $-240.000$ ↑ ↑ ↑ ...

Utbytter: $+30.000$, $+30.000 \cdot 1{,}04$, $+30.000 \cdot 1{,}04^2$

Nåverdi av utbytter:

$$\frac{30.000}{1+r} + \frac{30.000 \cdot 1{,}04}{(1+r)^2} + \frac{30.000 \cdot 1{,}04^2}{(1+r)^3} + \dots \quad \text{geometrisk}$$

$$k = \frac{1{,}04}{1+r} < 1 \quad \text{hvis } r > 4\%$$

$$= \underbrace{\frac{30000}{1+r}}_{a_1} \cdot \frac{1}{1 - \frac{1{,}04}{1+r}}$$

$$= \frac{30.000}{(1+r)-(1{,}04)} = \underbrace{\frac{30.000}{r-0{,}04}}_{\text{nåverdi utbytte}} = \underline{240.000} \quad \text{investering}$$

$$\frac{30.000}{240.000} = r - 0{,}04$$

$$0{,}125 = r - 0{,}04 \longrightarrow r = 0{,}165 = \underline{16{,}5\%}$$

C

5. $x^5 + 2x^4 = x + 2 \longleftarrow x = -2$ er løsn: $\begin{cases} VS: -32 + 2 \cdot 16 = 0 \\ HS: -2 + 2 = 0 \end{cases}$

$\Downarrow$

$x^5 + 2x^4 - x - 2 = 0 \longleftarrow (x+2)$ er faktor

$$(x^5 + 2x^4 - x - 2) : (x+2) = x^4 - 1$$
$$-(x^5 + 2x^4)$$
$$-x - 2$$
$$-x - 2$$
$$0$$

$x^5 + 2x^4 - x - 2 = 0$

$(x+2)(x^4 - 1) = 0$

$\underline{x = -2}$ eller $x^4 = 1$

$x^2 = \pm 1$

$x^2 < 0$ umulig

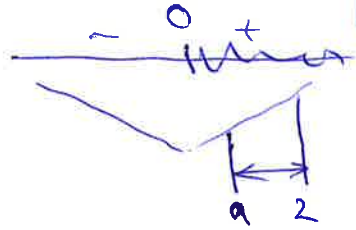
$x^2 = 1$

$\underline{x = \pm 1}$

C

6. $f'(x) = \underbrace{3\cdot(x^2-1)^2}_{\geq 0} \cdot \underbrace{2x}_{\substack{\geq 0 \text{ for } x \geq 0 \\ \leq 0 \text{ " } x \leq 0}}$

Fortegn for $f'(x)$




$a > 0$: $[a, 2]$ er intervall der $f$ er voksende

A

7. $f'(x) = 12x^2 - 2ax = 0$ for $x = 1/2$

$12 \cdot (\tfrac{1}{2})^2 - 2 \cdot a \cdot \tfrac{1}{2} = 0$

$3 - a = 0$

$\underline{a = 3}$

$f'(x) = 12x^2 - 2ax = \underline{12x^2 - 6x}$ med $a = 3$

$f'(x) = 0$ gir $12x^2 - 6x = 0$

$6x(2x - 1) = 0$

$\underline{x = 0}$ eller $\underline{x = 1/2}$

C

8. $(x^3 - 4x + 1) : (2x^3 - 5) = \frac{1}{2}$

$-(x^3 - 5/2)$

$-4x + 7/2 \leftarrow$ Rest

$f(x) = \frac{1}{2} + \frac{-4x + 7/2}{2x^3 - 5}$

$y = 1/2$ er horisontal asymptote

B

9. $f'(x) = \frac{1}{x\sqrt{x^2+1}} \cdot \left(x\sqrt{x^2+1}\right)' = \frac{1}{x\sqrt{x^2+1}} \cdot \left(1 \cdot \sqrt{x^2+1} + x \cdot \frac{1}{2\sqrt{x^2+1}} \cdot 2x\right)$

$= \frac{\sqrt{x^2+1} + x^2 \cdot \frac{1}{\sqrt{x^2+1}}}{x\sqrt{x^2+1}} \cdot \frac{\sqrt{x^2+1}}{\sqrt{x^2+1}} = \frac{(x^2+1) + x^2}{x(x^2+1)}$

$f'(2) = \frac{5+4}{2 \cdot 5} = \frac{9}{10} = \underline{0{,}9}$

A

10. $f'(x) = 2 \cdot \frac{1}{2\sqrt{x}} - \frac{1}{2}\left(x^{-1/2}\right)'$

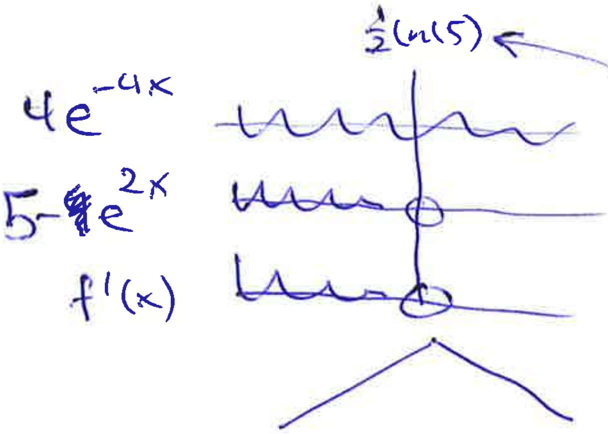
$$= \frac{1}{\sqrt{x}} - \frac{1}{2}\left(-\frac{1}{2}\right)x^{-3/2} = \frac{1}{\sqrt{x}} + \frac{1}{4}x^{-3/2}$$

$$f'(1) = \frac{1}{1} + \frac{1}{4}\cdot 1 = \underline{5/4} > 0$$

C

11. $f'(x) = 2e^{-2x}(-2) - 5e^{-4x}(-4)$

$$= -4e^{-2x} + 20e^{-4x} = 4e^{-4x}(5 - e^{2x})$$




$5 - e^{2x} = 0$

$e^{2x} = 5$

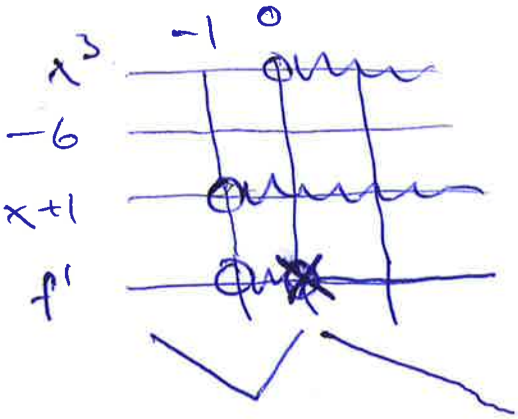
$2x = \ln 5$

$\underline{x = \frac{1}{2}\ln 5}$

$x = \frac{1}{2}\ln 5$ lokalt maks
ingen lokale min

A

12. $f' = -\frac{6}{x^2} + 3\cdot\left(\frac{-2}{x^3}\right) = \frac{-6x-6}{x^3} = \frac{-6(x+1)}{x^3}$




f voksende på $[-1, 0)$
f avtagende på $(-\infty, -1]$ og $(0, \infty)$

A

13. $D'(p) = \dfrac{-30\cdot p^2 - (200-30p)\cdot 2p}{p^4} = \dfrac{-30p^2 - 400p + 60p^2}{p^4}$

$= \dfrac{30p - 400}{p^3} \Rightarrow D'(5) = \dfrac{30\cdot 5 - 400}{5^3} = \dfrac{-250}{125} = -2$

$p=5$: $El_p\, D(p) = D'(p)\cdot \dfrac{p}{D(p)} = (-2)\cdot\dfrac{5}{D(5)} = \dfrac{-10}{2} = \underline{-5}$

$D(5) = \dfrac{200-30\cdot 5}{5^2} = \dfrac{50}{25} = 2$

C

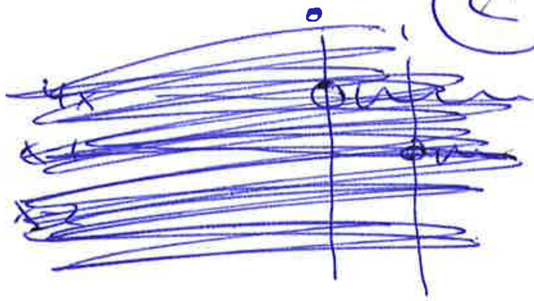
14. $\lim\limits_{x\to 1} \dfrac{x\sqrt{x}-1}{\ln x} \underset{\text{"0/0"}}{=} \lim\limits_{x\to 1} \dfrac{(x^{3/2})'}{1/x} = \lim\limits_{x\to 1} \dfrac{\frac{3}{2}x^{1/2}}{1/x} = \dfrac{\frac{3}{2}\cdot 1}{1} = \underline{\dfrac{3}{2}}$

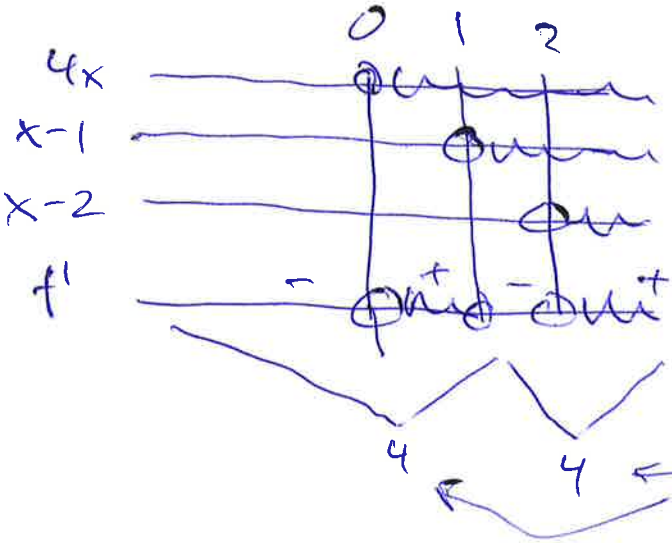
C

15. $f'(x) = 4x^3 - 12x^2 + 8x$

$= 4x\cdot(x^2-3x+2)$

$= 4x(x-1)(x-2)$






$f(0) = \underline{4}$

$f(2) = 16 - 4\cdot 8 + 4\cdot 4 + 4$

$= \underline{4}$

Globalt min:

$x=0$ og $x=2$ med min. verdi $= 4$

Globalt maks

Finnes ikke siden

$\lim\limits_{x\to\infty} f(x) = \infty$

($x^4$ dominerende)

D

(5 · 12 = 60 mnd)

16. Etter 5 år:

lånet vokst til $150.000 \cdot 1{,}0020^{60} = L$

$$A = \frac{L \cdot r(1+r)^n}{(1+r)^n - 1} = \frac{150.000 \cdot 0{,}0020 \cdot (1{,}0020)^{60} \cdot 1{,}0020^{180}}{1{,}0020^{180} - 1}$$

std. formel for annuitetslån med

$L = 150.000 \cdot 1{,}0020^{60} \simeq 169.104{,}26$

$r = 0{,}0020$

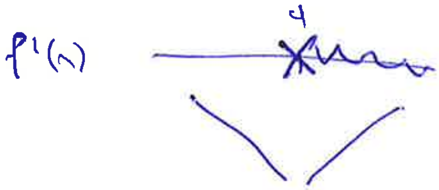
$n = 15 \cdot 12 = 180$

$A \simeq 1.119{,}62$ B

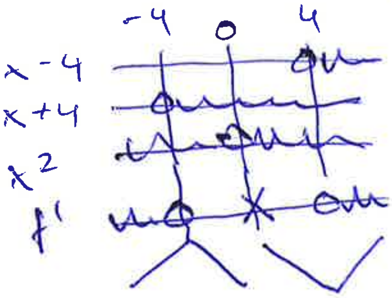
17. $f'(x) = 3 \cdot \frac{2}{3}(x-4)^{-1/3} = \frac{2}{\sqrt[3]{x-4}}$ ← pos.

← pos $x > 4$
neg $x < 4$

B



18. $f'(x) = 1 - \frac{16}{x^2} = \frac{x^2-16}{x^2} = \frac{(x-4)(x+4)}{x^2}$



lokalt min: $x = 4$
lokalt max $x = -4$

C

19. Fortegnslinja for $f'(x)$: Se

$x = 4$ lokalt min ($f$ er deriv. for $x = 4$, $f'$ ikke deriv. i $x = 4$)

ingen lokale maks

A