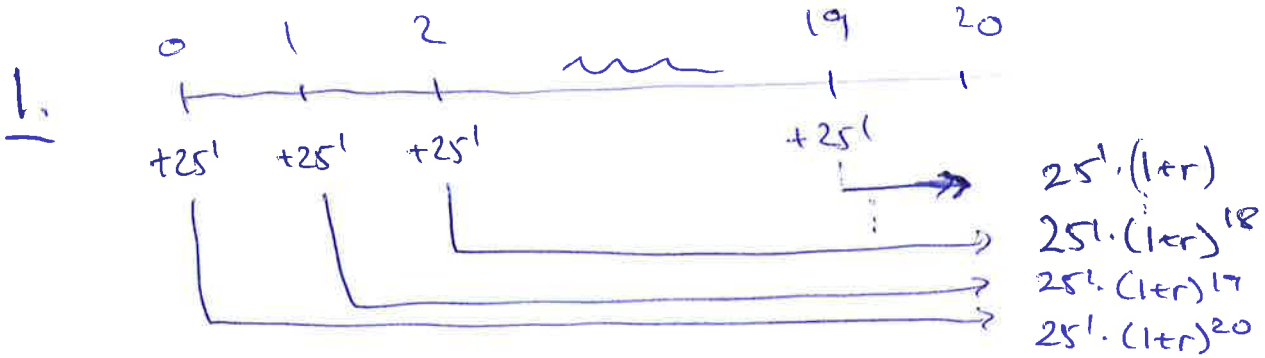


Løsning, prøve-eksamen April/2016

MET1180



Balanse på kto etter 20 år:

$$25000 \cdot 1,024 + 25000 \cdot 1,024^2 + \dots + 25000 \cdot 1,024^{20}$$

$$= 25.000 \cdot 1,024 \cdot \frac{1,024^{20} - 1}{1,024 - 1} \approx 647.400,58$$

Innskudd: $20 \cdot 25.000 = 500.000,00$

Samlende renter:

147.400,58

(C)

2. Samme oppsett som i 1:

$$25000 \cdot 1,05 \cdot \frac{1,05^{20} - 1}{1,05 - 1} \approx 867.981,30$$

- Innskudd:

$500.000,00$
367.981,30 kr

(D)

4. $k = \frac{1+g}{1+r}$ uendelig geometrisk rekke

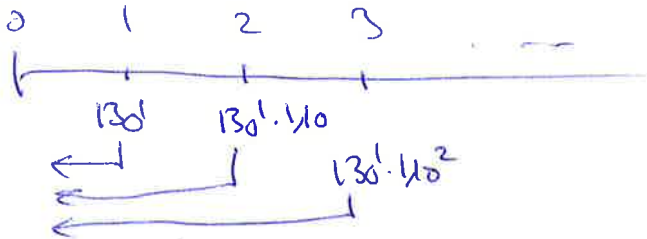
konvergent for $-1 < \frac{1+g}{1+r} < 1$

konv. for $g = 0,05 = 5\%$ \rightarrow $-(1+r) < 1+g < 1+r$ \rightarrow $g < r = 8\%$

$g = 0,05$: $a_1 \cdot \frac{1}{1-k} = \frac{10.000}{1,08} \cdot \frac{1}{1 - 1,05/1,08} = \frac{10.000}{0,08 - 0,05} \approx 333.333$

(B)

3.



$$\text{Nåverdi: } \frac{130.000}{1,08} + \frac{130.000 \cdot 1/10}{1,08^2} + \dots$$

$$= \frac{130.000}{1,08} \cdot \frac{1}{1 - \frac{1/10}{1,08}} = \frac{130.000}{0,08 - 0/10}$$

konverger ikke siden

$$k = \frac{1/10}{1,08} > 1$$

Nåverdi = ∞

(D)

5.

$$x^3 - 3x^2 + 4x = 2$$

$$x^3 - 3x^2 + 4x - 2 = 0$$

$x = \pm 1, \pm 2$ mulige heltallsloeser.

$x = 1: 0 = 0$ ok.

poly. div.

$$x^3 - 3x^2 + 4x - 2 = (x-1) \cdot (x^2 - 2x + 2) = 0$$

$x = 1$ eller $x^2 - 2x + 2 = 0$

$$x = \frac{2 \pm \sqrt{4 - 8}}{2} \text{ ingen løsn.}$$

$x = 1$ eneste løsn.

(A)

6.

~~$$\frac{3x-13}{4+\sqrt{x}} = 2$$~~

~~$$3x-13 = 2 \cdot (4+\sqrt{x}) = 8 + 2\sqrt{x}$$~~

~~$$3x-21 = 2\sqrt{x}$$~~

~~$$(3x-21)^2 = 4x$$~~

~~$$9x^2 - 30x + 25 = 4x$$~~

~~$$9x^2 - 34x + 25 = 0$$~~

~~$$x = \frac{34 \pm \sqrt{34^2 - 4 \cdot 9 \cdot 25}}{2 \cdot 9} = \frac{34 \pm 16}{18} = 1, \frac{50}{18} = \frac{25}{9}$$~~

Setter prøve: $x = \frac{25}{9}$: $VS = \frac{25}{3} - 5 = \frac{10}{3}$
 $HS = 2\sqrt{1/3} = 1,93$ ok
 $x = \frac{25}{9}$ eneste løsn.

(D)

$x = 1$ eller $x = \frac{25}{9}$

$$\underline{6.} \quad \frac{3x-13}{4+\sqrt{x}} = 2$$

$$3x-13 = 2(4+\sqrt{x}) = 8+2\sqrt{x}$$

$$3x-21 = 2\sqrt{x}$$

$$(3x-21)^2 = 4x$$

$$9x^2 - 126x + 441 = 4x$$

$$9x^2 - 130x + 441 = 0$$

$$x = \frac{130 \pm \sqrt{130^2 - 4 \cdot 9 \cdot 441}}{2 \cdot 9}$$

$$= \frac{130 \pm \sqrt{1024}}{18} = \frac{130 \pm 32}{18}$$

$$\underline{x=9} \quad \text{eller} \quad \underline{x = \frac{49}{9}}$$

Prøve: $3x-21 = 2\sqrt{x}$

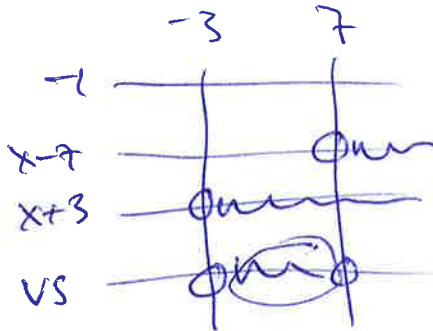
x=9 eneste
løsning.

x=9: VS: $3 \cdot 9 - 21 = \underline{6}$
HS: $2\sqrt{9} = 2 \cdot 3 = \underline{6}$ ok.

Ⓚ

x = $\frac{49}{9}$: VS: $\frac{49}{9} \cdot 3 - 21 = \frac{49}{3} - \frac{63}{3} = \underline{-\frac{14}{3}}$
HS: $2\sqrt{\frac{49}{9}} = \frac{2 \cdot 7}{3} = \underline{\frac{14}{3}}$ } falsk.

7. $20 + 4x - x^2 > -1$ $-x^2 + 4x + 21 = 0$
 $-x^2 + 4x + 21 > 0$ $x = \frac{-4 \pm \sqrt{16 + 84}}{-2} = 2 \pm \frac{10}{2} = \underline{7}, \underline{-3}$
 $-(x-7)(x+3) > 0$



Løsning: (-3, 7)

$x > 0$: $x < 7$

(B)

8. $f = x^4 - 4x^2 + 3 = 0$
 $x^2 = \frac{4 \pm \sqrt{16 - 4 \cdot 3}}{2} = \frac{4 \pm 2}{2} = 3, 1$
 $x^2 = 3$ eller $x^2 = 1$
 $x = \pm\sqrt{3}$ $x = \pm 1$
 $x_1 x_2 x_3 x_4 = \sqrt{3}(-\sqrt{3}) \cdot 1 \cdot (-1) = \underline{3}$

(D)

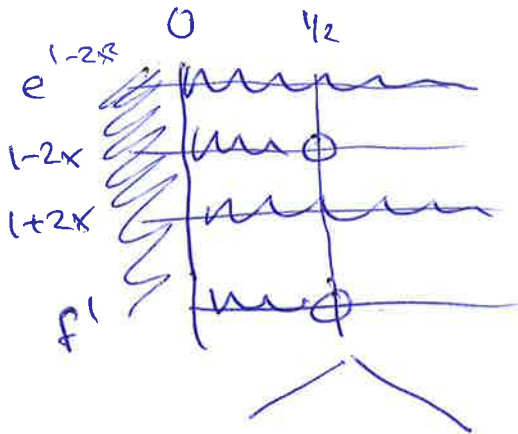
9. $f = \frac{x^2 - 6x + 8}{x^3 - x}$

Vertikale as: $x^3 - x = 0$
 $x(x^2 - 1) = 0$
 $x = 0, x = 1, x = -1$
 teller $\neq 0$ ($x = 0, \pm 1$)
3 vertikale
 $x_1 = 0, x_2 = 1, x_3 = -1$
 $a_1 + a_2 + a_3 = 0 + 1 - 1 = \underline{0}$

Horisontale as:
 $\frac{x^2 - 6x + 8}{x^3 - x} = 0 + \frac{x^2 - 6x + 8}{x^3 - x}$
 $y = 0$
 $b = 0$
 $a_1 + a_2 + a_3 = b$

(A)

10. $f = x e^{1-2x^2}, x \geq 0$
 $f' = 1 \cdot e^{1-2x^2} + x \cdot e^{1-2x^2} \cdot (-4x)$
 $= (1-4x^2) e^{1-2x^2} = (1-2x)(1+2x) e^{1-2x^2}$



voksende : $[0, 1/2]$
 avtagende : $[1/2, \rightarrow)$

(C)

11. $f = 3x^2 \ln(x^2 + x + 1)$
 $f' = 6x \cdot \ln(x^2 + x + 1) + 3x^2 \cdot \frac{1}{x^2 + x + 1} \cdot (2x + 1)$
 $f(1) = 6 \cdot \ln(3) + 3 \cdot \frac{1}{3} - 3 = \underline{6 \ln 3 + 3}$

(D)

12. $El_p D(p) = \frac{P}{D(p)} \cdot D'(p)$
 $= \frac{P}{\frac{2}{p^2} - \frac{3}{p^3}} \cdot (2p^{-2} - 3p^{-3})' = \frac{p \cdot (-4p^{-3} + 9p^{-4})}{\frac{2}{p^2} - \frac{3}{p^3}}$

$= \frac{\frac{-4}{p^2} + \frac{9}{p^3} \cdot p^3}{\frac{2}{p^2} - \frac{3}{p^3} \cdot p^3} = \frac{-4p + 9}{2p - 3} = \frac{9}{-3}$

$-4p + 9 = -(2p - 3)$
 $-4p + 2p = 3 - 9$
 $-2p = -6$
 $\underline{\underline{p = 3}}$

(C)

13. $\lim_{x \rightarrow 1} \frac{\ln x}{x^2 - 4x + 3} \stackrel{0/0}{=} \lim_{x \rightarrow 1} \frac{1/x}{2x - 4} = \frac{1}{-2} = \underline{\underline{-\frac{1}{2}}}$ (D)

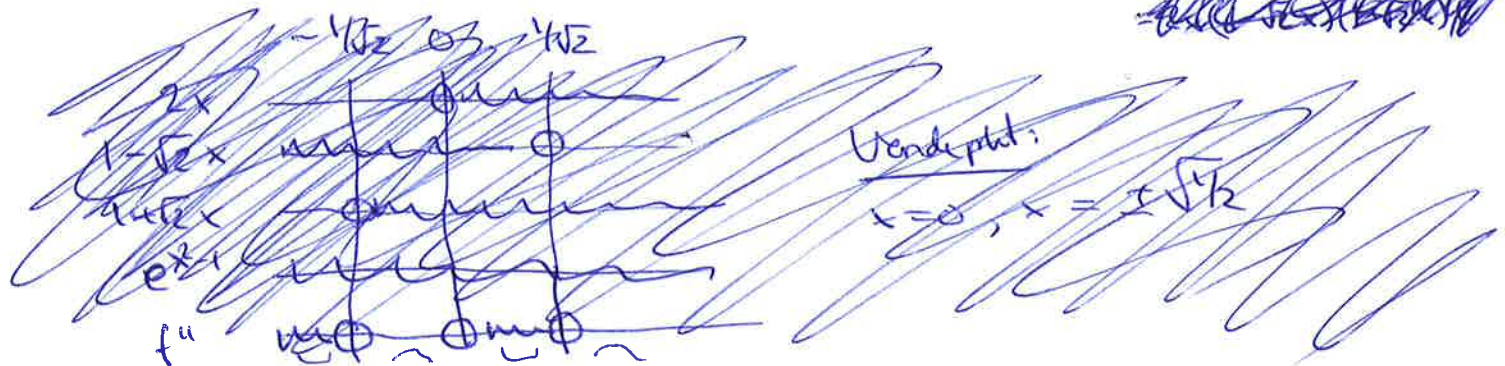
14. $f = (1-x)\sqrt{x^2+1}$
 $f' = -1 \cdot \sqrt{x^2+1} + (1-x) \cdot \frac{1}{2\sqrt{x^2+1}} \cdot 2x$
 $= -\sqrt{x^2+1} + \frac{x(1-x)}{\sqrt{x^2+1}} = \frac{-\sqrt{x^2+1} \cdot \sqrt{x^2+1} + x - x^2}{\sqrt{x^2+1}}$
 $= \frac{-(x^2+1) + x - x^2}{\sqrt{x^2+1}} = \frac{-2x^2 + x - 1}{\sqrt{x^2+1}}$

$-2x^2 + x - 1 = 0$
 $x = \frac{-1 \pm \sqrt{1-9}}{-4}$
 ingen løsn.
 \parallel
 $-2x^2 + x - 1$ neg.
 for alle x

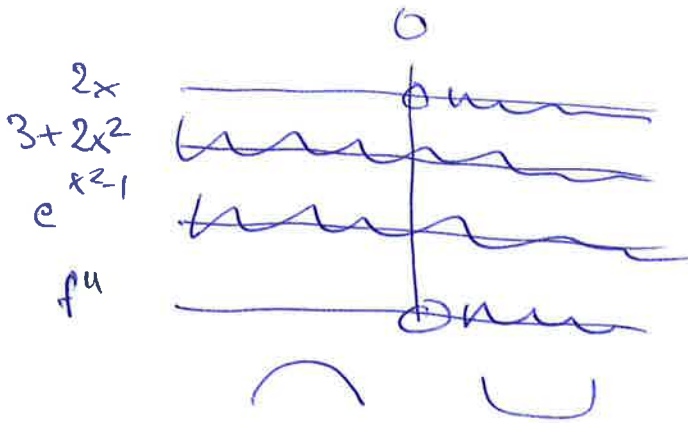
$f'(x) < 0$ for alle x
 ingen max/min

(A)

15. $f = x \cdot e^{x^2-1}$
 $f' = 1 \cdot e^{x^2-1} + x \cdot e^{x^2-1} \cdot 2x = (1+2x^2)e^{x^2-1}$
 $f'' = 4x \cdot e^{x^2-1} + (1+2x^2)e^{x^2-1} \cdot (2x)$
 $= [4x + 2x(1+2x^2)] e^{x^2-1} = 2x(2+1+2x^2) e^{x^2-1} = 2x(3+2x^2) e^{x^2-1}$



$$f''(x) = 2x(3+2x^2)e^{x^2-1}$$



Verdelykt:
 $x=0$

