Problem Sheet 4
DRE 7007 Mathematics

BI Norwegian Business School

Problems

1. Show that the open ball $B(p,r) \subseteq \mathbb{R}^n$ is convex for any point $p \in \mathbb{R}^n$ and every radius $r > 0$.

2. Determine if the functions are convex or concave:

a) $f(x,y) = e^{xy} - 1$
b) $f(x,y,z) = xyz$
c) $f(x,y,z) = \frac{1}{xyz}$

3. The Cobb-Douglas function $f : D \to \mathbb{R}$ defined on $D = \{(x,y) \in \mathbb{R}^2 : x,y \geq 0\}$ is given by

$$f(x,y) = Cx^a y^b$$

with $a,b,C > 0$. Compute the Hessian of f, and determine when it is convex and when it is concave.

4. Prove that the set $D = \{(x,y) \in \mathbb{R}^2 : x^2y^3 \geq 1, x > 0, y > 0\}$ is a convex set. Find the point in D closest to $(0,0)$, and use this to find a hyperplane that separates D and the point $(0,0)$.

Keep answers as short and to the point as possible. Answers must be justified.