BI NORWEGIAN BUSINESS SCHOOL

Written examination:	**DRE 70171 Mathematics, Ph.D.**	
Examination date:	17.09.2014 09:00 – 12:00	Total no. of pages: 2
Permitted examination support material:	A bilingual dictionary and BI-approved calculator TEXAS INSTRUMENTS BA II Plus	
Answer sheets:	Squares	
	Counts 100% of DRE 7017	The subquestions have equal weight
Ordinary exam		Responsible department: Economics

Question 1.

We consider the system of linear differential equations given by

$$\begin{aligned} \dot{x} &= 7x + 4y + 1 \\ \dot{y} &= 4x + y - 2 \end{aligned}$$

(a) Find the steady state $(\overline{x}, \overline{y})$.
(b) Rewrite the systen in the form $\mathbf{z}' = A\mathbf{z}$ and use this to solve the system.
(c) Find all initial states (x_0, y_0) such that $(x, y) \to (\overline{x}, \overline{y})$ when $t \to \infty$.

Question 2.

We consider the function $f(x, y, z, w) = x^2 + y^2 + z^2 + w^2 + xw - yz$ defined on $\mathbb{R}^4$.

(a) If f convex? Is it concave?
(b) Find the global maximum and minimum values of f, if they exist.

Question 3.

For positive constants $a, b, T > 0$, we consider the optimal control problem

$$\max \int_0^T \ln(ax - bu)\, \mathrm{d}t \quad \text{subject to} \quad \begin{cases} x(0) = 1 \\ x(T) = \frac{1}{2}\, e^{aT/b} \\ x' = u \\ u \in U \end{cases}$$

with control region $U = \mathbb{R}$.

(a) Show that the function $(x, u) \mapsto \ln(ax - bu)$ is concave.
(b) Solve the optimal control problem.

Question 4.

We consider the functions $f_n(x) = x^n/n$ for $n = 1, 2, 3, \ldots$ in the function space $V = C([0,1])$ of continuous functions on the unit interval $[0,1]$. We equip V with the sup norm

$$||f|| = \sup_{x \in [0,1]} |f(x)|$$

and the corresponding metric $d(f,g) = ||f - g||$ for $f, g \in V$.

(a) Compute $||f_1||$, $||f_2||$ and $d(f_1, f_2)$.
(b) Compute $d(f_n, f_{n+k})$.
(c) Is (f_n) a Cauchy sequence in V? Find the limit of the sequence, if it exists.