Exercise Problems

Problem 1.
Let A be the 3×4 matrix with parameter a given by

$$A = \begin{pmatrix} 1 & 1 & 2 & 1 \\ 3 & 2 & 1 & 0 \\ 2 & 5 & a & 11 \end{pmatrix}$$

Show that $\mathbf{v} = (2, -3, 0, 1)$ is in the vector space $V = \text{Null}(A)$, and find a base of V containing $\mathbf{v}$.

Problem 2.
Compute all eigenvalues and eigenvectors for the following $n \times n$ matrices, and find an orthonormal base of $\mathbb{R}^n$ consisting of eigenvectors when this is possible:

a) $A = \begin{pmatrix} 2 & -3 \\ 7 & -8 \end{pmatrix}$ b) $A = \begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}$ c) $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$

Problem 3.
Determine if the matrices are positive (semi)definite, negative (semi)definite or indefinite:

a) $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 4 & 5 \\ 0 & 5 & 8 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & a \end{pmatrix}$ c) $\begin{pmatrix} 1 & -2 & -1 & 1 \\ -2 & 1 & 1 & 2 \\ -1 & 1 & -1 & -3 \\ 1 & 2 & -3 & 0 \end{pmatrix}$

Problem 4.
Write the following dynamical system (in discrete time) in matrix form:

$$\begin{aligned} x_{t+1} &= 0.75x_t + 0.35y_t \\ y_{t+1} &= 0.25x_t + 0.65y_t \end{aligned}$$

We assume that the initial state (x_0, y_0) satisfies $x_0 + y_0 = 1$. Does the system tend towards an equlibrium in the long run (as $t \to \infty$)? If so, what is the equilibrium state?

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