Exercise Problems

Problem 1.
Solve the following optimal control problem for $T = 3$ using dynamic programming:

$$\max \sum_{t=0}^{T} (3 - u_t)x_t^2$$

subject to $x_{t+1} = u_t x_t$ and $x(0) = 1$ when the control region $U = [1,3] \subseteq \mathbb{R}$.

Problem 2.
Consider the optimal control problem

$$\max \sum_{t=0}^{\infty} \beta^t \left(-\frac{4}{9}x_t^2 - u_t^2 \right)$$

subject to $x_{t+1} = x_t + u_t$ and $x(0) = x_0$ when the control region $U = \mathbb{R}$. Find a solution to the Bellman equation of the form $J(x) = -Ax^2$. Is it unique?