Exercise Problems

Problem 1.
Find all critical points of the function $f(x,y) = x^4 + 2x^2y^2 + y^4 - x^2 - y^2$ defined on $D = \mathbb{R}^2$, and classify them as local maxima, local minima or saddle points. Does f have a global maximum or minimum on D?

Problem 2.
Consider the function $f : D \to \mathbb{R}$, defined by $f(x,y) = \ln(1-x^2-y^2)$ on the open set $D = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 < 1\}$. Is f concave? Find $\max f(x,y)$ when $(x,y) \in D$.

Problem 3.
Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$, defined by $f(x,y) = 3x^4 + 3x^2y - y^3$. Find all the critical points of f and classify their type. Is there a global maximum or a global minimum for f?