Exercise Problems

Problem 1.
Find the steady state $(\overline{x}, \overline{y})$ and use it to solve the linear system of differential equations given by

$$\begin{aligned} x' &= x - 2y - 5 \\ y' &= x + 4y + 1 \end{aligned}$$

Find the initial states (x_0, y_0) such that $(x,y) \to (\overline{x}, \overline{y})$ as $t \to \infty$.

Problem 2.
Solve the linear system of differential equations given by

$$\begin{aligned} x' &= x - 2y - 6z + 7 \\ y' &= 2x + 5y + 6z - 4 \\ z' &= -2x - 2y - 3z + 4 \end{aligned}$$

Find the initial states (x_0, y_0, z_0) such that $(x,y,z) \to (\overline{x}, \overline{y}, \overline{z})$ as $t \to \infty$.

Problem 3.
In the paper *Innovation, Imitation, and Intellectual Property Rights*, Helpman considers the following non-linear systems of differential equations

$$\begin{aligned} \dot{\xi} &= g - (g+m)\xi \\ \dot{g} &= \left(\frac{L^N}{a} - g\right)\left[\rho + m + g - \frac{1-\alpha}{\alpha}\left(\frac{L^N}{a} - g\right)\frac{1}{\xi}\right] \end{aligned}$$

where m, L^N, a, ρ, α are positive parameters with $0 < \alpha < 1$. We consider values of the variables ξ, g with $0 < \xi < 1$ and $0 < g < L^N/a$. Express the linearized system of differential equations at the steady state in terms of $\overline{\xi}$ and $\overline{g}$, and show that the system is not globally asymptotically stable. (Hint: Use specific values for the parameters if you cannot solve the problem for general values).