Exercise Problems

Problem 1.
Let $S = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$, and let $T = \{(x,y) \in \mathbb{R}^2 : xy = 1\}$. Sketch the regions S and T in the plane, and find the boundaries ∂S and ∂T. For each of the regions, determine if it is open, closed, bounded, compact.

Problem 2.
We consider the function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} x \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Show that f is continuous at $x = 0$. Is f differentiable at $x = 0$? Is f a C^1 function? What about the function $g : \mathbb{R} \to \mathbb{R}$ given by

$$g(x) = \begin{cases} x^2 \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Problem 3.
We consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Show that f is a C^1 function, and compute its Hessian matrix. Is it a C^2 function?