Exercise Problems

Problem 1.
Show that the open ball $B(p,r) \subseteq \mathbb{R}^n$ is convex for any point $p \in \mathbb{R}^n$ and every positive radius $r > 0$.

Problem 2.
Determine if the functions are convex or concave:

a) $f(x,y) = e^{xy} - 1$ b) $f(x,y,z) = xyz$ c) $f(x,y,z) = \frac{1}{xyz}$

Problem 3.
Determine whether the functions are convex or concave, and also if they are quasi-convex or quasi-concave:

a) $f(x) = x^3$ b) $f(x) = \ln(x)$ c) $f(x) = e^{-x^2}$

Problem 4.
The Cobb-Douglas function $f : D \to \mathbb{R}$ defined on $D = \{(x,y) \in \mathbb{R}^2 : x, y \geq 0\}$ is given by $f(x,y) = Cx^a y^b$ where a,b and C are positive constants. Compute the Hessian of f, and determine when f is convex and when it is concave. What about quasi-convex or quasi-concave?

Problem 5.
Prove that the set $D = \{(x,y) \in \mathbb{R}^2 : x^2y^3 \geq 1, x > 0, y > 0\}$ is a convex set. Find the point in D closest to $(0,0)$, and use this to find a hyperplane that separates D and the point $(0,0)$.