

Exercise Problems

Problem 1.

Solve the optimal control problem

$$\max \int_0^2 (3 - x^2 - u^2) \, dt$$

subject to $x' = u$, $x(0) = 1$, $x(2) = 4$ when the control region $U = \mathbb{R}$.

Problem 2.

Solve the optimal control problem

$$\max \int_0^T \left(x - \frac{1}{2}u^2 \right) \, dt$$

subject to $x' = u$, $x(0) = x_0$ when the control region $U = \mathbb{R}$. Use the optimal pair (x^*, u^*) to compute the optimal value function

$$V(x_0, T) = \int_0^T \left(x^* - \frac{1}{2}(u^*)^2 \right) \, dt$$