EBA 2911, lecture 6, 18 Sept. 2019, Runar Ile

Plan:
1. Repetition & exercises
2. Polynomial division & factorisation
3. Rational- & radical equations
4. Inequalities

1. Rep. & exc.

Linear expression in standard form: $ax+b$

Quadratic expression on std. form: $a\cdot x^2+bx+c$

Linear equation: can be written as $ax+b=0$

Quadratic equation: — " — $ax^2+bx+c=0$

Quad. eq. have 2, 1 or no solutions

b^2-4ac pos, $=0$, neg.

Find solutions by applying the quadratic formula or by completing the square.

Prob. 4e) Solve the eq. $x^2\boxed{-24}x = -25$ by completing the square.

$:2$

we add $(-12)^2$ on both sides

Solution:
$$(x\boxed{-12})^2 = 25+(-12)^2$$
$$(x-12)^2 = 169$$

$x-12=\sqrt{169}=13$ or $x-12=-\sqrt{169}=-13$

that is: $x = \underline{\underline{25}}$ or $x=\underline{\underline{-1}}$

Factorisation and roots (zeros)

Prob. 3e) We know that $x^2+bx+c=0$ has the solutions $x = 3 \pm \sqrt{5}$. Then

$$x^2+bx+c = (x-\overbrace{(3-\sqrt{5})}^{r_1})\cdot(x-\overbrace{(3+\sqrt{5})}^{r_2})$$

$$= x^2-(3+\sqrt{5})x-(3-\sqrt{5})x+(3^2-(\sqrt{5})^2)$$

$$= x^2-(3+\cancel{\sqrt{5}}+3-\cancel{\sqrt{5}})x+4$$

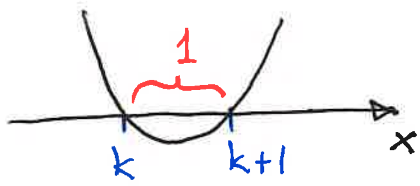
$$= \underline{\underline{x^2 \overbrace{-6}^{-(r_1+r_2)}x+\overbrace{4}^{r_1 \cdot r_2}}} \qquad (b=-6,\ c=4)$$

Parameters: numbers without explicit values

- used to describe many situations simultaneously

Ex: The price of a product is p kroner

Prob. 7a) All polynomials x^2+bx+c which have two zeros of distance 1 can be written as



$$\overbrace{(x-k)}^{\text{zero: } x=k}\cdot\underbrace{(x-(k+1))}_{\text{zero: } x=k+1}$$

where k is the smallest zero.

Then $(x-k)(x-(k+1)) = \underline{x^2-(2k+1)x+k(k+1)}$

2. Polynomial division

Want to divide a polynomial $f(x)$ with a polynomial $g(x)$ and get a polynomial $q(x)$ with a remainder $r(x)$

$$\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)} \quad \Big| \cdot g(x)$$

gives $$f(x) = q(x) \cdot g(x) + r(x)$$

Ex: $f(x) = 3x^2 + 2x + 1$

$g(x) = x - 2$

$3x^2 : x \quad\quad 8x : x$

$$\boxed{3x^2} + 2x + 1 : (\boxed{x} - 2) = 3x + 8 + \frac{17}{(x-2)}$$

answ. multiply multiply answer

$$\begin{array}{l} -(3x^2 - 6x) \\ \hline \quad \boxed{8x} + 1 \\ -(8x - 16) \\ \hline \quad\quad 17 \end{array}$$

17 the remainder (since $\deg(17) = 0 < 1 = \deg(x-2)$)

So $q(x) = 3x + 8$ and $r(x) = 17$

$$\left(3x+8+\frac{17}{(x-2)}\right)\cdot(x-2)$$

$$= 3x^2+8x-6x-16+\frac{17}{\cancel{(x-2)}}\cdot\cancel{(x-2)}$$

$$= 3x^2+2x+1 = f(x)$$

So $3x^2+2x+1 = (3x+8)(x-2)+17$

Two applications of polynomial division

A) To find asymptotes of rational functions.

$$\frac{3x^2+2x+1}{x-2} = 3x+8+\frac{17}{x-2}$$

has a vertical asymptote $x=2$

and a non-vertical asymptote $y=3x+8$

3) To factorise a polynomial as a product of degree 1 (linear) polynomials.

<u>Ex</u>: Factorise $x^3-4x^2-11x+30$ into linear factors.

Solution: Three steps.

I) Guess an integer solution (zero)

[Note: has to divide 30]

I try $x = -3$: $(-3)^3 - 4\cdot(-3)^2 - 11\cdot(-3) + 30$

$= -27 - 36 + 33 + 30 = 0$

Then $(x - (-3))$ is a factor in the polynomial!

II) Use polynomial division to factorise the polynomial as a product of $(x - (-3)) = (x+3)$ and a polynomial of degree 2.

$$\begin{array}{l} (\boxed{x^3} - 4x^2 - 11x + 30) : (\boxed{x} + 3) = \underset{x^3 : x}{x^2} \underset{-7x^2 : x}{- 7x} + \underset{10x : x}{10} \\ -(x^3 + 3x^2) \\ \hline \qquad \boxed{-7x^2} - 11x + 30 \\ \quad -(-7x^2 - 21x) \\ \hline \qquad\qquad \boxed{10x} + 30 \\ \qquad\quad -(10x + 30) \\ \hline \qquad\qquad 0 \text{ remainder} \end{array}$$

This means: $x^3 - 4x^2 - 11x + 30 = (x^2 - 7x + 10)(x+3)$

III) We find the roots (zeros) of $x^2 - 7x + 10$. They are $x = 2$, $x = 5$. So $x^2 - 7x + 10 = (x-2)\cdot(x-5)$

Then: $\underline{\underline{x^3 - 4x^2 - 11x + 30 = (x-2)(x-5)(x+3)}}$.

Note 1: Not always possible to factorise second degree polynomials

Ex: x^2+5 , x^2+2x+3

$b^2-4ac = 2^2-4\cdot1\cdot3 = 4-12<0$

Note 2: It can be difficult to guess a root – it doesn't have to be an integer.

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3. Rational- and radical equations

A rational equation: $\frac{p(x)}{q(x)} = 0$ $p(x)$ and $q(x)$ are polynomials

Ex 1: $\frac{x+1}{(x-1)(x+3)} = 0$ so $x+1=0$

(and $x \neq 1$, $x \neq -3$)

Ex 2: $\frac{x+1}{(x-1)(x+3)} = 2$

subtract 2 from b. sides.

$$\frac{x+1}{(x-1)(x+3)} - 2 = 0$$

multiply -2 with $\frac{(x-1)(x+3)}{(x-1)(x+3)}$

(which is 1)

$$\frac{x+1}{(x-1)(x+3)} - 2\cdot\frac{(x-1)(x+3)}{(x-1)(x+3)} = 0$$

$$\frac{x+1-2(x^2+2x-3)}{(x-1)(x+3)} = 0$$

$$\frac{x+1-2x^2-4x+6}{(x-1)(x+3)} = 0$$

$$\frac{-2x^2-3x+7}{(x-1)(x+3)} = 0$$

that is: $-2x^2-3x+7=0$

$(x \neq 1,\ x \neq -3)$

which you can solve.

Radical equations

- the unknown is under the root.

<u>Ex</u>: $2\sqrt{x+1} = x-2$

square b.s.

$$4\cdot(x+1) = x^2-4x+4$$

$$4x+\cancel{4} = x^2-4x+\cancel{4}$$

$$x^2-8x=0$$

$$x(x-8)=0$$

so $x = \underline{0}$ or $x = \underline{8}$

Note: Not all of these need to be solutions of the first equation.

We have to test the candidates:

$\underline{x=0}$ l.h.s $2\sqrt{0+1} = 2\cdot\sqrt{1} = 2\cdot 1 = 2$

r.h.s $0 - 2 = -2$

not equal, $x=0$ is not a solution

$\underline{x=8}$ l.h.s. $2\sqrt{8+1} = 2\sqrt{9} = 2\cdot 3 = 6$

r.h.s. $8 - 2 = 6$

equal and so $x = \underline{8}$ is the only solution.

4. Inequalities

$-2 < -1$ read: 'minus two is less than minus one'

$\frac{1}{9} > \frac{1}{12}$ read: 'one ninth is bigger than one twelfth'

All: $\leq$ and $\geq$

An inequality is a claim that one expression (number) is $>, <, \geq, \leq$ another expression (number)

The solutions of an inequality are those values of x which makes the claim true.

EX: $x - 1 \geq 2$ is a claim

- is true for $x = 5$ since $5 - 1 \geq 2$.
- is not true for $x = 2$ since $2 - 1 \geq 2$ is not true!

The solutions of the inequality are all the values of x such that

$$\underline{\underline{x \geq 3}}$$

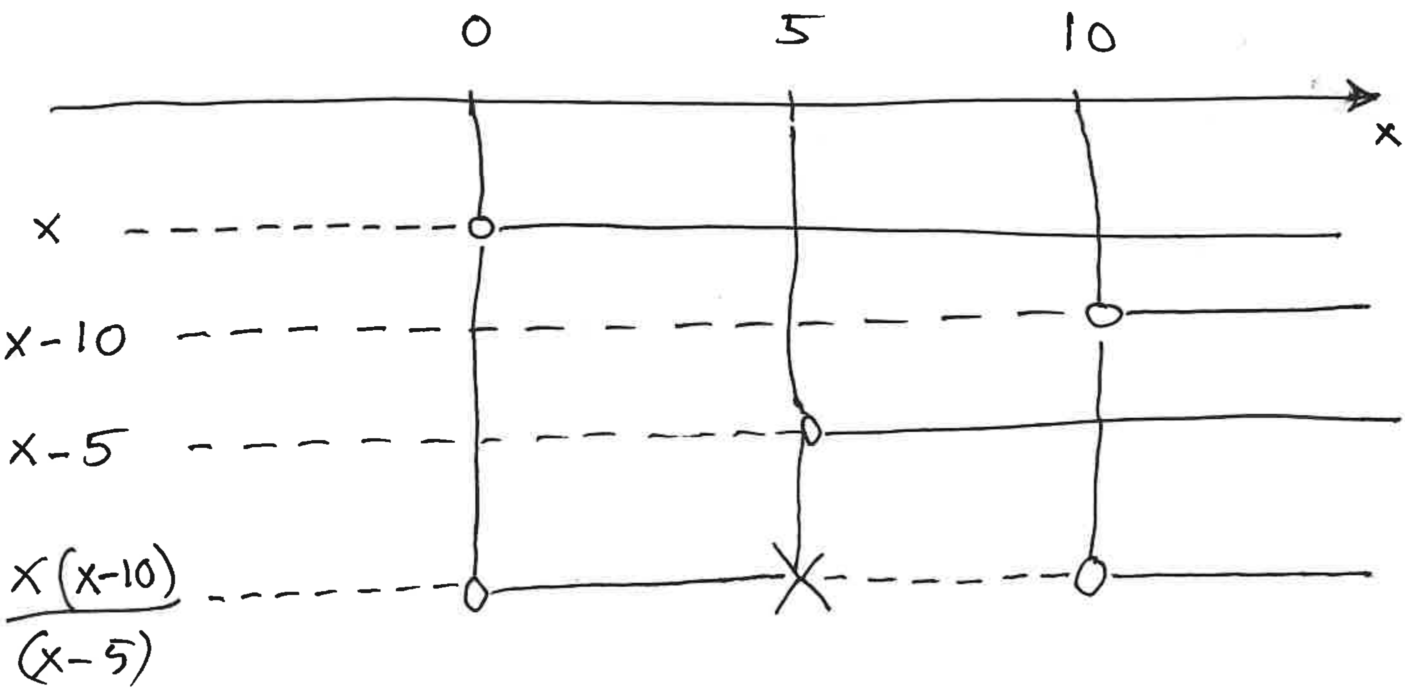
- an infinite set of numbers!

We also write $x \in [3, \infty\rangle$

$x \in [3, \rightarrow\rangle$

EX: Solve the inequality $\dfrac{x(x-10)}{(x-5)} \geq 0$

Solution: Since we have 0 on the r.h.s and a factorised fraction we can appy the sign diagram immedeately.



that is $\underline{0 \leq x < 5 \text{ or } x \geq 10}$

alternate way of writing: $\underline{x \in [0, 5\rangle \text{ or } x \in [10, \infty\rangle}$