EBA 2911, lecture 7, 25 Sept. 2019, Runar Ile

Plan:
1. Repetition & exercises
2. Functions and graphs
3. Linear functions & straight lines
4. Quadratic functions & parabolas

1. Rep. & exercises

Polynomial division $f(x) : g(x) = q(x) + \frac{r(x)}{g(x)}$

remainder

polynomials

Use:
- *) Determine asymptotes
- *) Factor polynomials

If the polynomial $f(x)$ has a root (a zero) k then $f(x) : (x-k) = q(x)$ – a polynomial of one degree less than $f(x)$

Prob. 2b $f(x) = x^3 + 6x^2 - x - 30$

Guess that $x=2$ is a zero: $f(2) = 2^3 + 6\cdot 2^2 - 2 - 30$
$= 8 + 24 - 32 = 0$

Then $x-2$ is a factor in $f(x)$.
We find $f(x) : (x-2)$ by polynomial division:

$$(x^3 + 6x^2 - x - 30) : (x-2) = x^2 + 8x + 15$$

$$\begin{array}{l}
-(x^3 - 2x^2) \\
\hline
\quad 8x^2 - x - 30 \\
\quad -(8x^2 - 16x) \\
\hline
\qquad 15x - 30 \\
\qquad -(15x - 30) \\
\hline
\qquad 0
\end{array}$$

0 ← remainder

so $x^3 + 6x^2 - x - 30$
$= (x^2 + 8x + 15)(x-2)$
roots: $-3, -5$
$= (x+3)(x+5)(x-2)$

Note: All the roots divide -30

- all integer zeros do that.

4th degree polynomial:

- Guess a (zero) r_1
- Use polynomial division $f(x) : (x - r_1)$ to find 3rd degree polynomial $q(x)$
- Guess a zero r_2 for $q(x)$
- Use polynomial division $q(x) : (x - r_2)$ to find a quadratic polynomial $p(x)$
- Factor $p(x)$ as a product of two linear polynomials.

Radical equations ('x is under the root')

Plan: Remove the square root by squaring on each side of the eq. But then the root has to isolated on one of the sides of the eq.

Prob. 5d $\sqrt{2x+1} - \sqrt{x+4} = 1$

Isolate one of the roots.

$$\sqrt{2x+1} = 1 + \sqrt{x+4}$$

square each side

$$2x+1 = 1^2 + 2 \cdot 1 \cdot \sqrt{x+4} + \left(\sqrt{x+4}\right)^2$$

$$2x + \cancel{1} = \cancel{1} + 2\sqrt{x+4} + x + 4$$

Isolate $2\sqrt{x+4}$ on the r.h.s.

$\boxed{\text{Quadratic exp: } (a+b)^2 = a^2 + 2ab + b^2}$

$$x-4 = 2\sqrt{x+4}$$

square each side

$$(x-4)^2 = 4(x+4)$$

use quadratic expression

$$x^2 + 2\cdot(-4)x + (-4)^2 = 4x+16$$

$$x^2 - 8x + \cancel{16} = 4x + \cancel{16}$$

$$x^2 - 12x = 0$$

$$x(x-12) = 0$$

so either $x=0$ or $x-12=0$

$x = 12$.

Have to test if these really are solutions of the original equation

$\underline{x=0}$ l.h.s $\sqrt{2\cdot 0+1} - \sqrt{0+4} = \sqrt{1} - \sqrt{4} = 1-2 = -1$

r.h.s. 1 – different, and $x=0$ is <u>not</u> a solution.

$\underline{x=12}$ l.h.s. $\sqrt{2\cdot 12+1} - \sqrt{12+4} = \sqrt{25} - \sqrt{16} = 5-4$

$= 1$

r.h.s. 1

– equal, and $x = \underline{12}$ is the only solution.

Inequalities

If we multiply (divide) with a negative number, the inequality flips.

Ex:
$$-4 < -3 \quad \Big| \cdot (-2)$$

$$8 = (-4)(-2) > (-3)\cdot(-2) = 6$$

Prob. 7j
$$\frac{(x-2)(x+3)}{(x-5)(x+4)} < 1$$

Plan: Subtract 1 from b.s. and write everything on the l.h.s. as one fraction.

$$\frac{(x-2)(x+3)}{(x-5)(x+4)} - 1 < 0$$

$$\frac{(x-2)(x+3)}{(x-5)(x+4)} - 1 \cdot \frac{(x-5)(x+4)}{(x-5)(x+4)} < 0$$

$$\frac{(x-2)(x+3) - (x-5)(x+4)}{(x-5)(x+4)} < 0$$

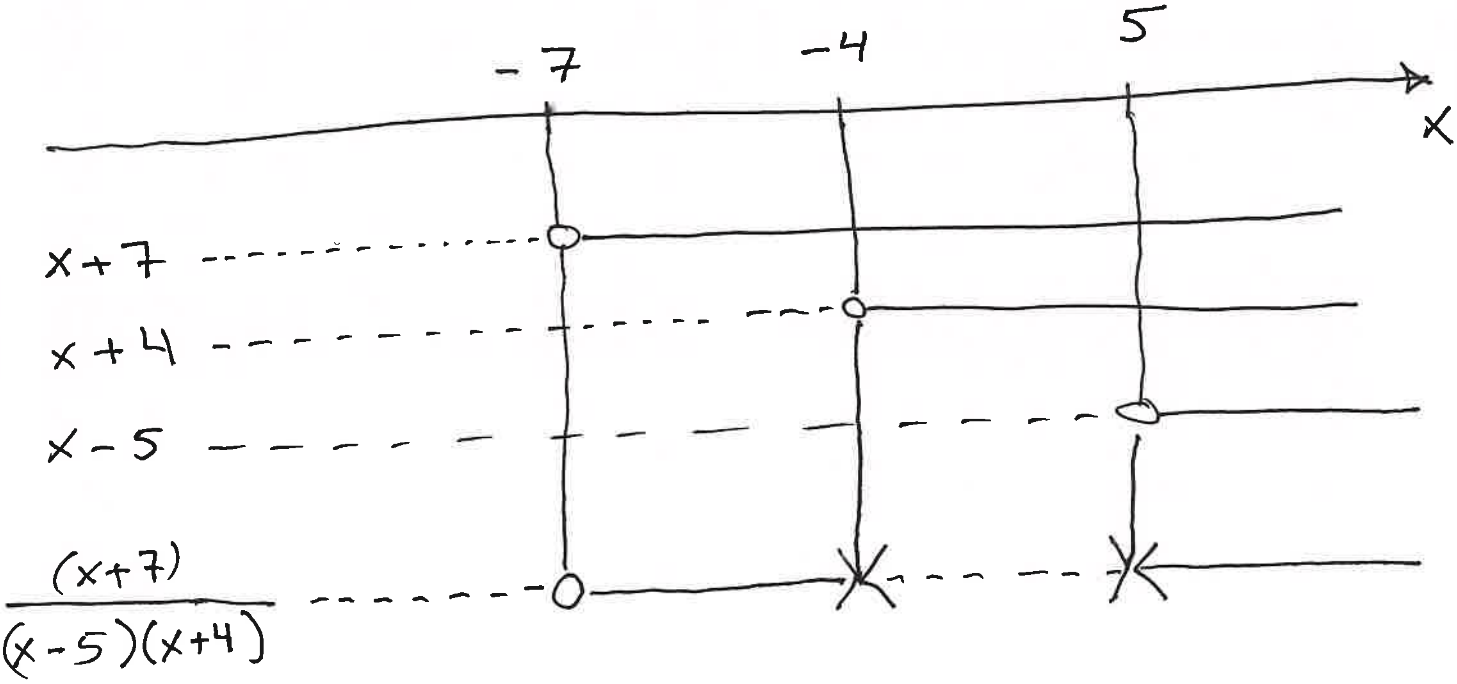
$$\frac{\cancel{x^2} + 3x - 2x - 6 - [\cancel{x^2} + 4x - 5x - 20]}{(x-5)(x+4)} < 0$$

$$\frac{2x+14}{(x-5)(x+4)} < 0 \quad | : 2$$

$2x + 14 = 2(x+7)$

$$\frac{(x+7)}{(x-5)(x+4)} < 0$$

Use a sign diagram:



Hence $x < -7$ or $-4 < x < 5$

Alternative way of writing:

$x \in \langle \leftarrow, -7) \cup \langle -4, 5)$

2. Functions & graphs

* function is a tabel of function values

x	. . . . .
$f(x)$	. . . . . .

<u>Ex</u>: Empirical functions

- the temperature as a function of time
- fertility
- the price of salmon
- all kinds of 'indexes' (CPI)

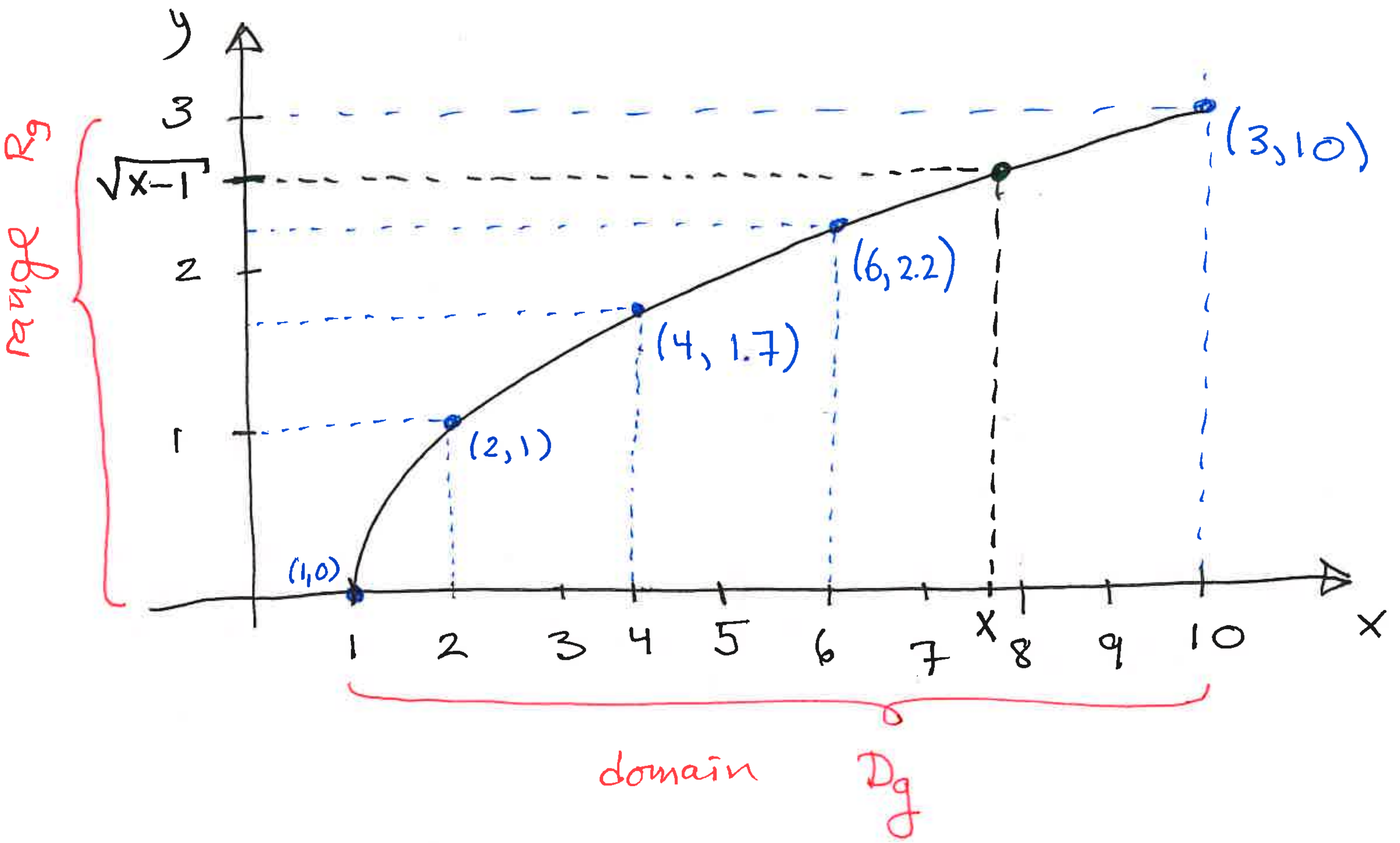
$f(x)$ = average age of first child birth in year x.

Domain of definition $x \in [1961, 2018]$

<u>Ex</u>: $g(x) = \sqrt{x-1}$. The largest possible domain of definition is $D_g = [1, \rightarrow\rangle$

Want to draw the graph with $D_g = [1, 10]$. I make a function value table

x	1	2	4	6	10
$g(x)$	0	1	1.7	2.2	3



3. Linear functions $\quad f(x) = ax + b$

– the graph is a line

The point-slope-formula

(x_0, y_0) is a point on the graph

and s is the slope, then

$$y - y_0 = s(x - x_0)$$

Ex if $(x_0, y_0) = (9, 25)$

$(x_1, y_1) = (11, 31)$ are two

points on the line, then the slope

is $s = \dfrac{31 - 25}{11 - 9} = \dfrac{6}{2} = 3$

the point-slope formula gives

$$y - 25 = 3(x-9)$$

so $$y = 3x - 27 + 25$$

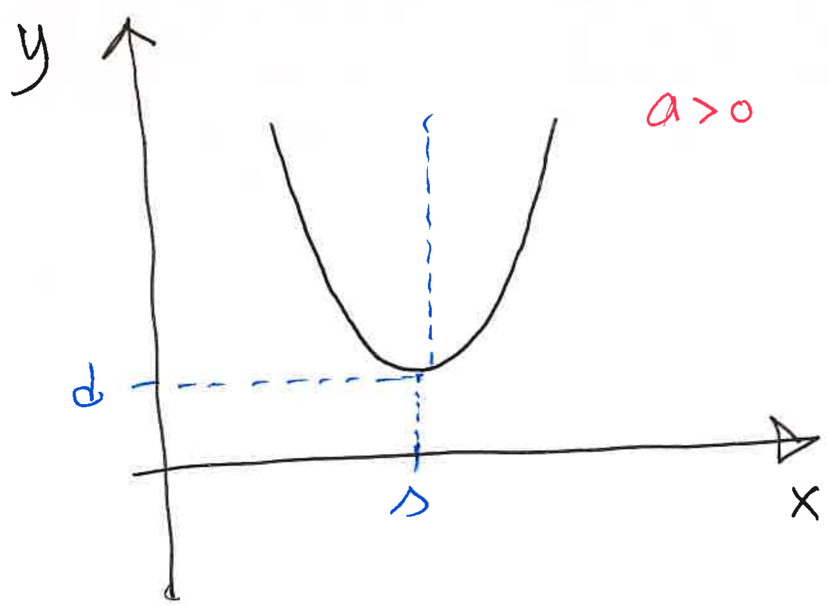
$$y = 3x - 2$$

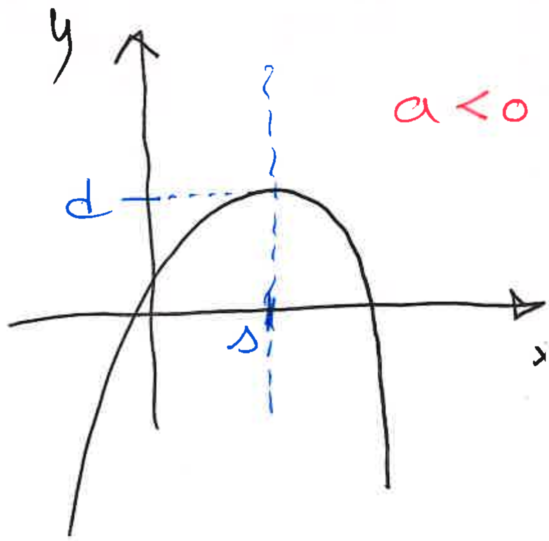
4. Quadratic functions & parapolas.

$$f(x) = ax^2 + bx + c$$

But if we want draw the graph the following expression is better:

$$f(x) = a(x-s)^2 + d$$ ("by completing the square")





Ex: $f(x) = x^2 - 2x + 3$
$= (x-1)^2 + 2$ (so $s=1$, $d=2$)