EBA 2911, lecture 8, 2 oct 2019, Runar Ile

1. Problems from the execise session
2. Increasing and decreasing functions
3. Circles and ellipses
4. Polynomial functions

1. Problems

Prob. 3a Because we have two easy-to-read zeros, we use the form $f(x) = a(x - r_1)(x - r_2) = a(X-2)(x-5)$

the roots

$f(0)=5$ so $a(0-2)(0-5) = 5$

$$a \cdot 10 = 5$$

$$a = \frac{5}{10} = \frac{1}{2} = 0.5$$

and so $f(x) = \frac{1}{2}(x-2)(x-5)$

3b) $x = 2$ is the larger root,

$x = -\frac{1}{2}$ is the symmetry axis,

so the smaller root has to be

$$x = -\frac{1}{2} - 2{,}5 = -3 .$$

Hence $f(x) = a(x-2)(x+3)$

$f(0) = 6$, $a(0-2)(0+3) = 6$

$$a \cdot (-6) = 6$$

$$a = \frac{6}{-6} = -1$$

and so $f(x) = -(x-2)(x+3)$

3c) We see that $x = 100$ is a double root, so

$$f(x) = a(x-100)(x-100) = a(x-100)^2$$

Since $(80, 40)$ is a point on the graph,

$$f(80) = 40, \quad \text{so} \quad a(80-100)^2 = 40$$

$$a \cdot (-20)^2 = 40$$

$$a \cdot 400 = 40$$

$$a = \frac{40}{400} = \frac{1}{10} = 0.1$$

and so $\underline{\underline{f(x) = \frac{1}{10}(x-100)^2}}$

3d) We observe the symmetry axis $x = 1$

and the maximum value $y = -1$

Then $f(x) = a(x-s)^2 + d$

$$= a(x-1)^2 - 1$$

since $f(0) = -2$, we get $a(0-1)^2 - 1 = -2$

$$a - 1 = -2$$

$$a = -2+1 = -1$$

and $\underline{\underline{f(x) = -(x-1)^2 - 1}}$

3e) The symmetry axis is $x = -3$

The minimum value is $y = 4.25$

so $f(x) = a(x+3)^2 + 4.25$

from $f(-2) = 4.5$ we get $a \cdot (-2+3)^2 + 4.25 = 4.5$

$$a + 4.25 = 4.5$$

$$a = 0.25$$

and $f(x) = 0.25(x+3)^2 + 4.25$

7a) Three points on the graph: $P=(0,7)$

No extra info. $Q=(1,4)$

so I use the form $R=(2,3)$

$f(x)=ax^2+bx+c$.

P: $f(0)=a\cdot 0^2+b\cdot 0+c=7$

$\underline{c=7}$.

Q: $f(1)=a\cdot 1^2+b\cdot 1+7=4$

$\boxed{a+b=-3}$ (1)

R: $f(2)=a\cdot 2^2+b\cdot 2+7=3$

$\boxed{4a+2b=-4}$ (2)

Solve this system of equations

From (1) we get $4a+4b=-12$ (mult. (1) by 4)

subtract (2): $\underline{4a+2b=-4}$

$0a+2b=-12-(-4)=-8$

$b=\frac{-8}{2}=\underline{-4}$

From (1) we get $a=-3-(-4)=\underline{1}$

so $f(x)=\underline{\underline{x^2-4x+7}}$

Summary (second degree polynomial functions)

3 standard forms:

A) If we know the roots: $f(x) = a(x-r_1)(x-r_2)$

B) If we know the symmetry axis $x = s$

and the max/min value $y = d$

then $f(x) = a(x-s)^2 + d$

C) Other cases: $f(x) = ax^2 + bx + c$

(but we can always use B)

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2. Increasing and decreasing functions

Ex: $f(x) = 0.03x^2 + 8x - 1500$, $D_f = [0, \rightarrow\rangle$

(meaning: $x \geq 0$)

Is $f(x)$ increasing?

Is $f(x)$ decreasing?

or neither?

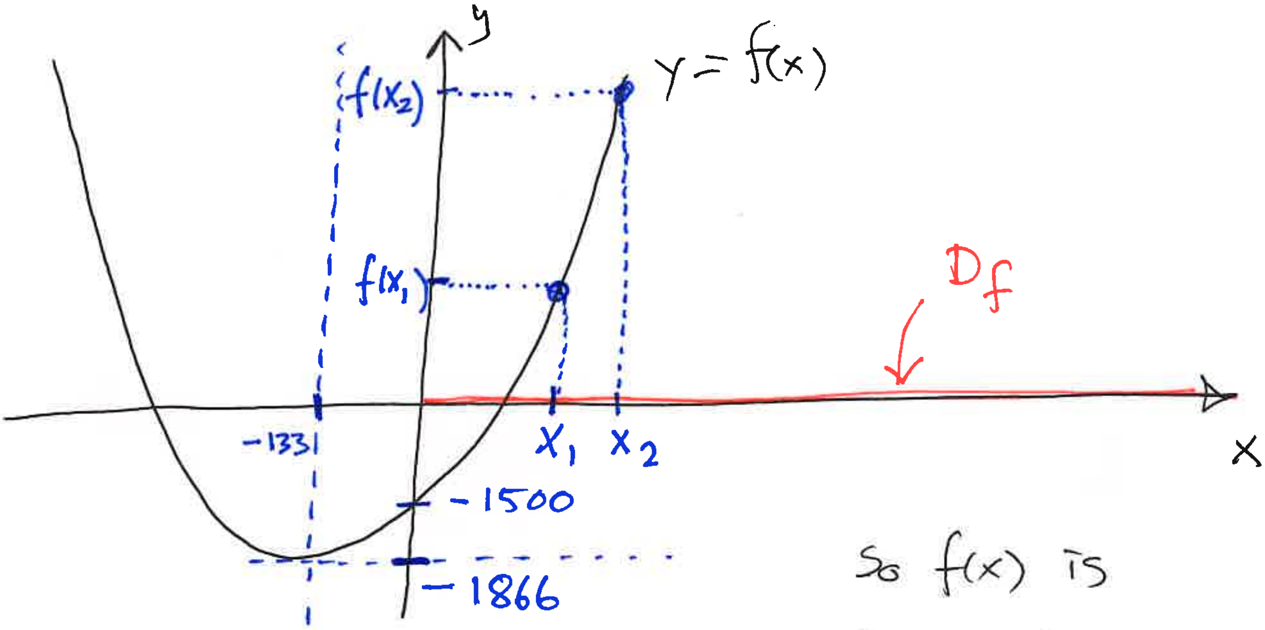
- can look at the graph (use GeoGebra or similar)

or: complete the square and draw the graph by hand.

Get: $f(x) = 0.03\left(x + \frac{800}{6}\right)^2 - \frac{5600}{3}$

Symmetry axis: $x = -\frac{800}{6} \approx -133$

Minimum value: $y = -\frac{5600}{3} \approx -1866$



So $f(x)$ is increasing when $x \geqslant 0$.

<u>Definition</u> A function $f(x)$ is <u>increasing</u> if for all $x_1 < x_2$ one has

$$f(x_1) \leqslant f(x_2)$$

<u>Ex</u>: $f(x) = 2x + 5$ is increasing for all x:

Reason: If $x_1 < x_2$

multiply by 2 on each side

$$2x_1 < 2x_2$$

add 5 to each side

$$f(x_1) = 2x_1 + 5 < 2x_2 + 5 = f(x_2)$$

so $f(x)$ is (strictly) increasing.

Definition A function $f(x)$ is decreasing if for all $x_1 < x_2$ one has
$$f(x_1) \geq f(x_2)$$

Problem: Show that $f(x) = -2x + 5$ is decreasing.

Solution: suppose $x_1 < x_2 \quad | \cdot (-2)$

$$-2x_1 > -2x_2$$

- add 5 to each side

$$f(x_1) = -2x_1 + 5 > -2x_2 + 5 = f(x_2)$$

and so $f(x)$ is decreasing.

Problem : We have the constant function $f(x) = 5$

Decide whether $f(x)$ is increasing, decreasing or neither.

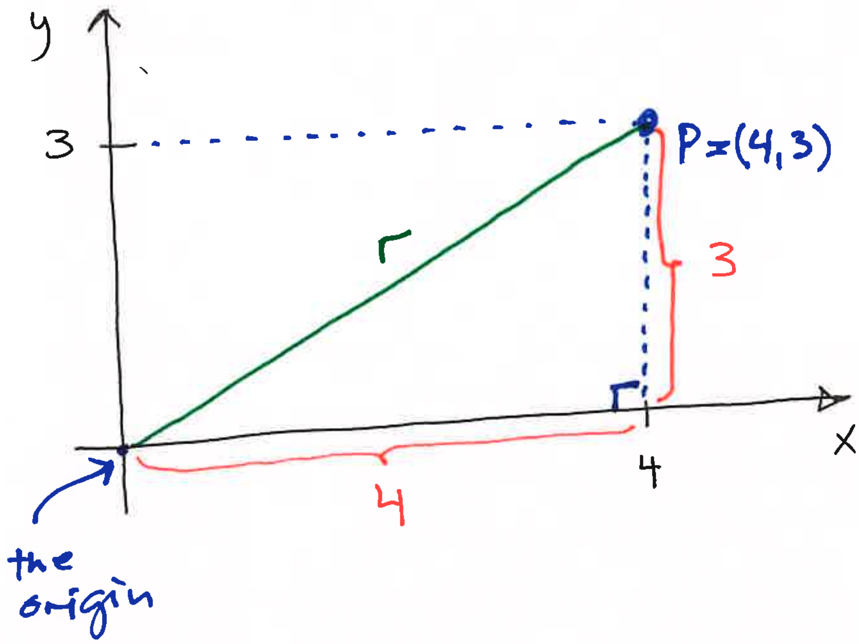
Increasing: If $x_1 < x_2$ then $f(x_1) = 5 \leq 5 = f(x_2)$

Decreasing: If $x_1 < x_2$ then $f(x_1) = 5 \geq 5 = f(x_2)$

Definition $f(x)$ is strictly increasing if $f(x_1) < f(x_2)$ for all $x_1 < x_2$.

$f(x)$ is strictly decreasing if $f(x_1) > f(x_2)$ for all $x_1 < x_2$.

3. Circles and ellipses



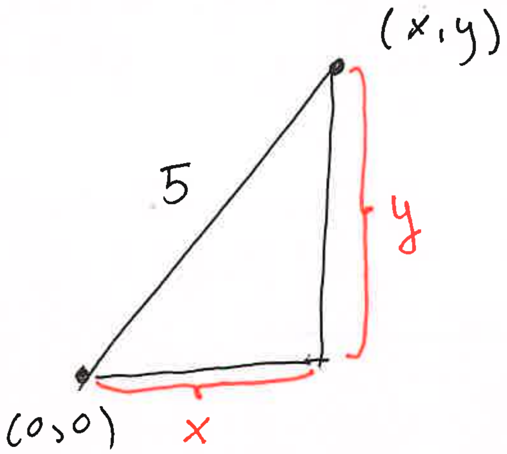
What is the distance from P to the origin?

Pythagoras gives the answer:

$$r^2 = 4^2 + 3^2 \quad (r \geq 0)$$

$$r^2 = 16 + 9 = 25$$

$$\underline{\underline{r = 5}} \quad (= \sqrt{25})$$



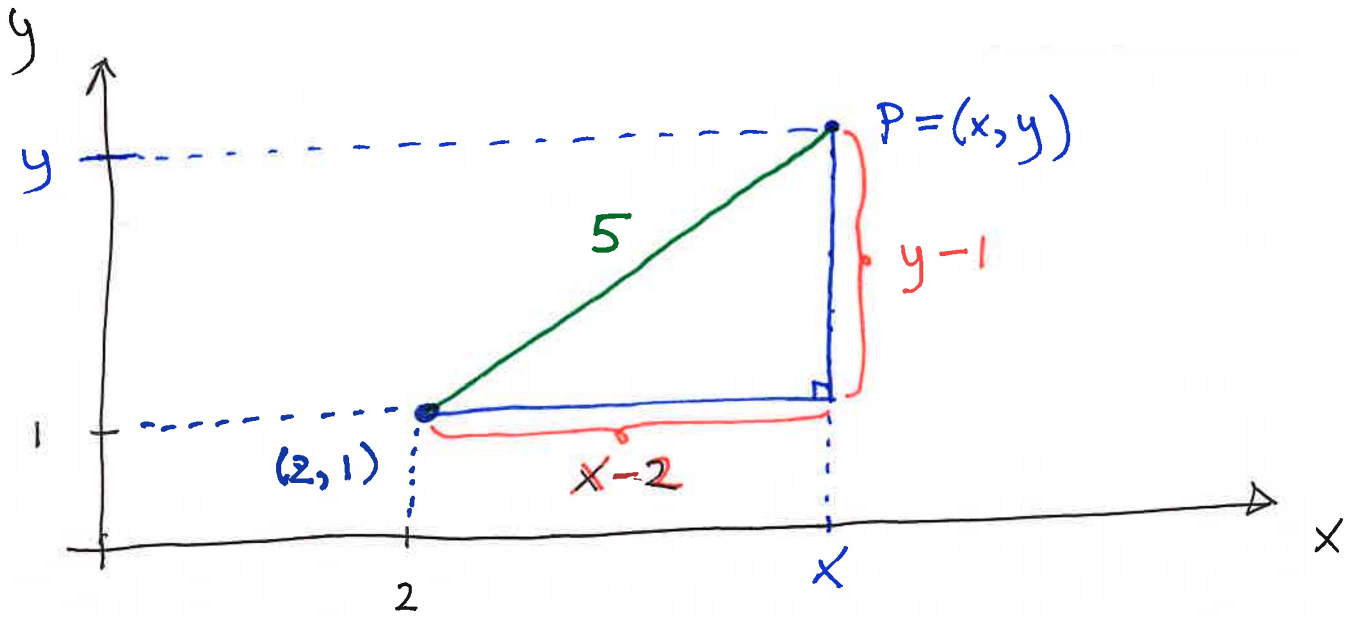
Pythagoras:

$$5^2 = x^2 + y^2$$

- one equation, two unknowns
- infinitly many solutions

The solutions are all the points on the circle with the origin as centre and with radius 5.

Ex: What is the equation of the points on the circle with (2, 1) as centre and radius 5?



Pythagoras: $5^2 = (x-2)^2 + (y-1)^2$

$$25 = x^2 - 4x + 4 + y^2 - 2y + 1$$

that is $x^2 + y^2 - 4x - 2y = 20$

<u>Problems</u> Determine the radius and the centre of the circle.

a) $(x-3)^2 + (y-2)^2 = 16$

b) $x^2 + (y+5)^2 = 10$

c) $x^2 + y^2 - 2x + 6y = -9$

<u>Solutions:</u>

a) centre is $\underline{(3,2)}$, radius is $\sqrt{16} = \underline{4}$

b) centre $\underline{(0,-5)}$, radius $= \underline{\sqrt{10}}$

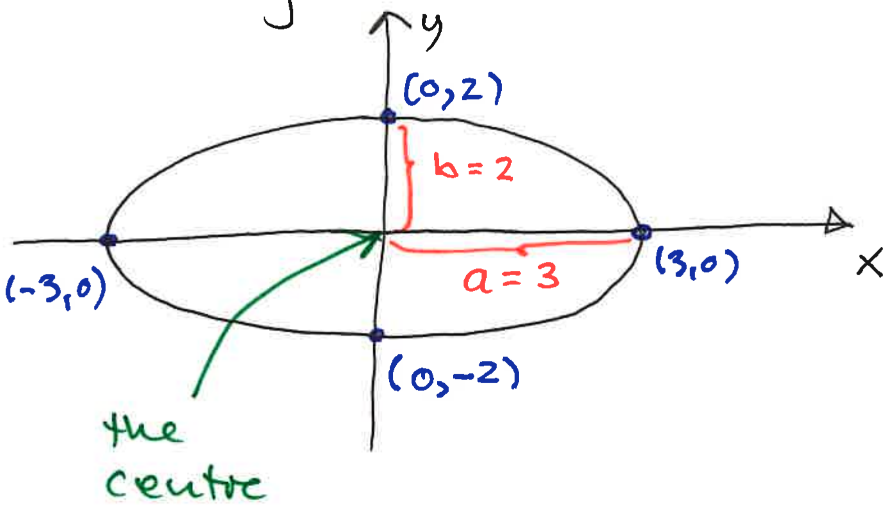
c) $\underbrace{(x-1)^2}_{x^2-2x+1} + \underbrace{(y+3)^2}_{y^2+6y+9} = -9 + 1^2 + 3^2 = 1$

centre: $\underline{(1,-3)}$, radius $= \sqrt{1} = \underline{1}$.

Ellipses

x	3	-3	0	0
y	0	0	2	-2

Ex : $4x^2 + 9y^2 = 36$



Divide the eq. by 36 :

$$\underbrace{\frac{4}{36}}_{=\frac{1}{9}} \cdot x^2 + \underbrace{\frac{9}{36}}_{=\frac{1}{4}} \cdot y^2 = 1$$

$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$$

reminds us of the circle equation, but the x-axis is streched by a factor 3 and the y-axis —— " —— 2.

In general, any ellipse is the set of solutions of an equation

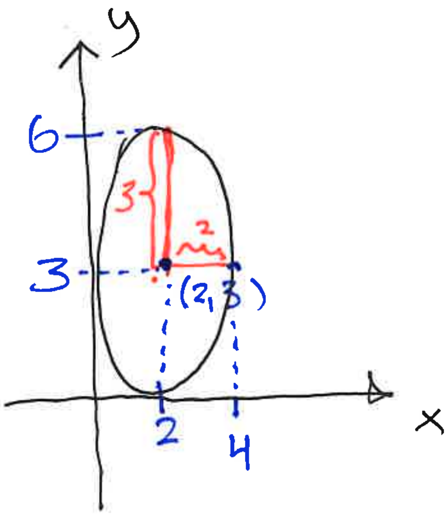
$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$$

Here (x_0, y_0) is the centre and a and b are the semi-axes

Ex: $$\frac{(x-2)^2}{4} + \frac{(y-3)^2}{9} = 1$$

centre: $(2, 3)$

semi-axes $\quad a = \sqrt{4} = 2$, $\quad b = \sqrt{9} = 3$



4. Polynomial functions

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$$

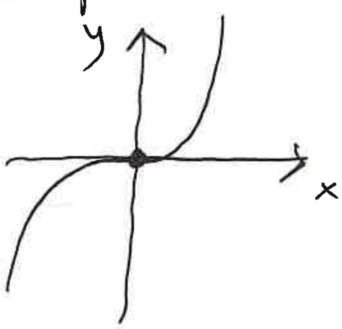
$$(a_n \neq 0)$$

is a polynomial function of degree n.

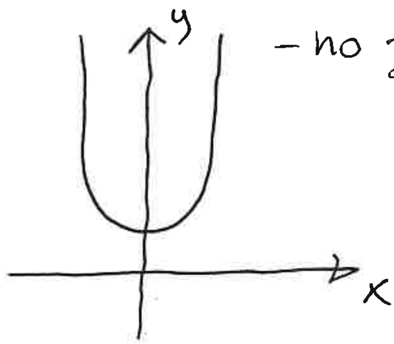
It has at maximum n zeros (roots)

If n is odd it has at least one zero.

Ex: $f(x) = x^3$



Ex: $f(x) = x^4 + 1$



- no zeros

$\underline{\underline{Ex}}$: $f(x) = (x-1)^2(x-2)$ is a third degree polynomial function with two zeros : $x = 1$, $x = 2$